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KINEMATICS AND DYNAMICS OF ALTERNATIVE CONVERTING MECHANISM FOR PERSPECTIVE ENGINES

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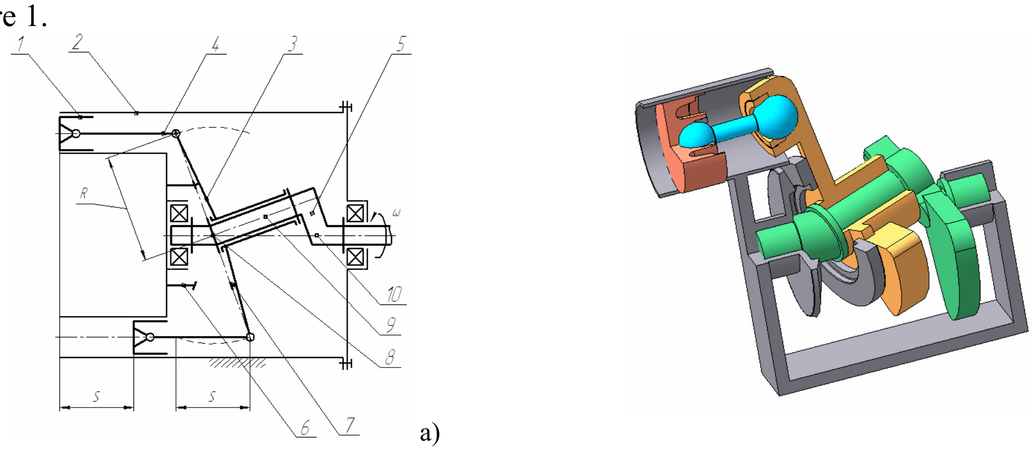
ABSTRACT - The present research work is aimed at the investigation on cone pendulum (CP) mechanism for converting the crank motion into the rotation that can be applied to such machines as internal combustion engines, pumps and compressors. The paper describes cone pendulum, its kinematics characteristics such as the trajectory, the acceleration, the velocity and the movement of any point of the body determining the CP motion process. The mathematical description of these parameters and special cases of CP motion are considered in depth. The dynamic compensation together with compensation conditions are discussed in relation to engines with cone pendulum mechanism. The work presents also the advantages and the examples of CP application.
The depth analysis of kinematics and dynamics of the developed converting mechanism allows to find its basic features and to accent nuances as applied to axial-piston engines. In the conclusion the paper gives the simulation results for an engine with cone pendulum as applied to racing cars.

TECHNICAL PAPER - The axial-piston engines (APE) can be considered as real alternative to other commonly used motor concepts as V- or in-line engines. The advantages of APE lie in very compact cylindrical design, smaller weight as against to comparable conventional motors and bi-bearing crankshaft regardless of cylinders number. Also it is possible to decrease pressure on the cylinder's walls to change the compression ratio and cylinder stroke capacity. Axial-piston engines are characterized by motion converting mechanisms, which are essentially alternative to the common crank mechanisms.
All investigation results presented in this work go into problems on a cone pendulum (CP) as the base of the novel converting mechanism. CP in the form of a swing plate is shown on Figure 1.



a) b)

Figure 1: Cone pendulum as converting mechanism: a) Schematic diagram; b) 3D-model

In this mechanism the piston 1 in the cylinder 2 has a kinematic join with the swing plate 3 through the rod 4. The plate 3 can turn out of position of the crankshaft 5. The conic gear 6

united with engine block and the conic gear 7 united with the swing plate 3 ensure the stabilization effect. Figure 1 displays also the center 8 of the converting mechanism, the prime axis 9 and the crank axis 10. The piston stroke length is defined as follows: $S=2\cdot R\cdot sin\alpha$, where α is the angle between the prime axis 9 and crank axis 10.
It is necessary to understand that the cone pendulum is a solid body performing the precession motion (5). The precession motion is a regular repeating motion that is defined by a fixed point and an axle, which is passing through this point shaping a cone surface in space. With reference to the presented converting mechanism the swing plate is the cone pendulum performing the precession (spherical) motion. The plane, which is perpendicularly to the prime axis passing through the fixed point of the swing plate, is further termed the equatorial plane (EP).
For descriptive purposes, the paper compares further the developed 3-cylinder APE with the "cone pendulum" converting mechanism and the typical inclined engine for racing cars and bikes (analog of Yamaha FZ6-S engine). Both engines have similar power parameters, equal cylinder capacity and compression ratio. The simplified view of designed 3-cylinder APE is presented on Figure 2. Data of the inclined engine (1, 3, 4) and the designed APE engine are resulted in the Table 1.

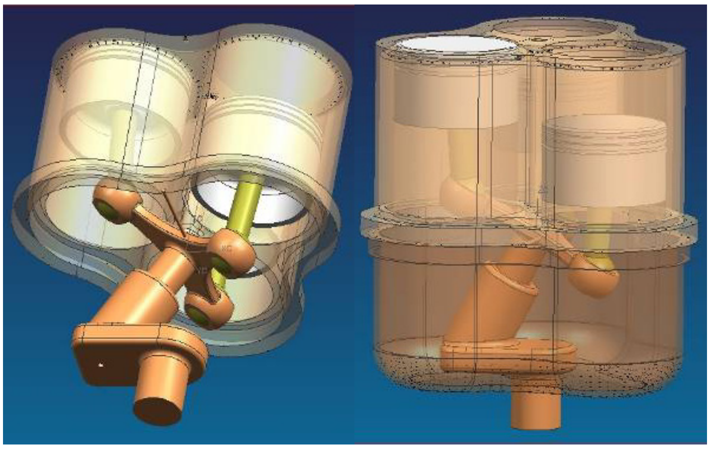


Figure 2: Sketch of 3-cylinder axial-piston engine

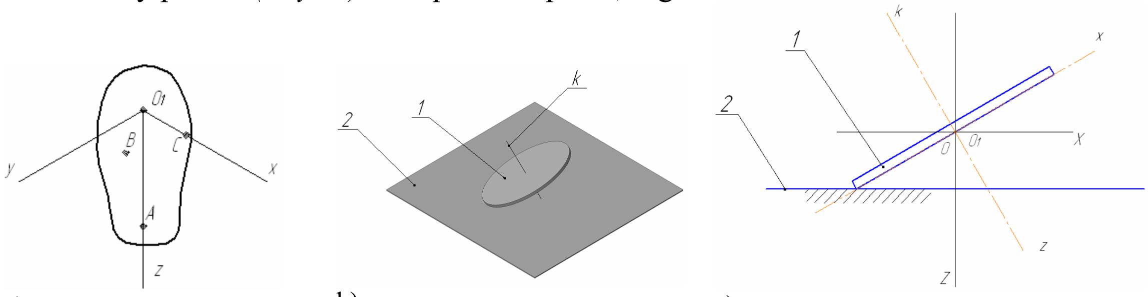
Table 1: Engine data

	Inclined engine (analog of Yamaha FZ6-S engine)	3-cylinder APE under discussion
Engine type	Liquid-cooled, 4 stroke	Liquid-cooled, 4 stroke
Displacement	600 cm^3	590 cm^3
Cylinder arrangement	Forward-inclined parallel 4-cylinder	Axial 3-cylinder
Bore × stroke	65.5 × 44.5	80 × 39.2
Rod length	91.5 mm	60 mm
Compression ratio	12.2:1	12:1
Power	98 hp at 12000 rpm	94 hp at 12000 rpm

KINEMATICS

General theory

The main tasks of kinematics of a cone pendulum are the definition of the motion law and the investigation on such characteristics as the trajectory, the velocity, the acceleration and the movement for any point of the body determining the CP motion. Let us consider the solid in the coordinate system $xyzO_1$ to determine the CP kinematic characteristics. Therefore any point of the solid is characterized by the coordinates (x,y,z). The next discussion deals with the point $A(0,0,z_A)$ on the prime axis 0_1z, the point $C(x_C,0,0)$ on the equatorial plane $xy0$, and an arbitrary point $B(x_B,y_B,0)$ on equatorial plane, Fig. 3a.



a) b) c)

Figure 3: Kinematic diagrams

There are a number of limitations for the motion of solid: the point 0_1 is rigidly bound with the coordinate system $XYZ0$, the axis 0_1z shapes a cone surface by the movement and can deviate from the axis $0Z$ on a fixed angle α at any moment.

For example, the CP motion process can be compared with the disk *1*, which runs on the flat surface *2* without slipping, Figure 3b. Here the *k* is the prime axis, and the equatorial plane is the plane of disk *1* contacting with the flat surface *2* during the motion. The projection of the disk *1* on the plane $XZ0$ is presented in Figure 3c.

The movement of the point A$(0,0,x_A)$ along the axes $X0$, $Y0$ and $Z0$ can be calculated by as follows

$$A_X = z_A \sin\alpha \cos\varphi , \tag{1}$$

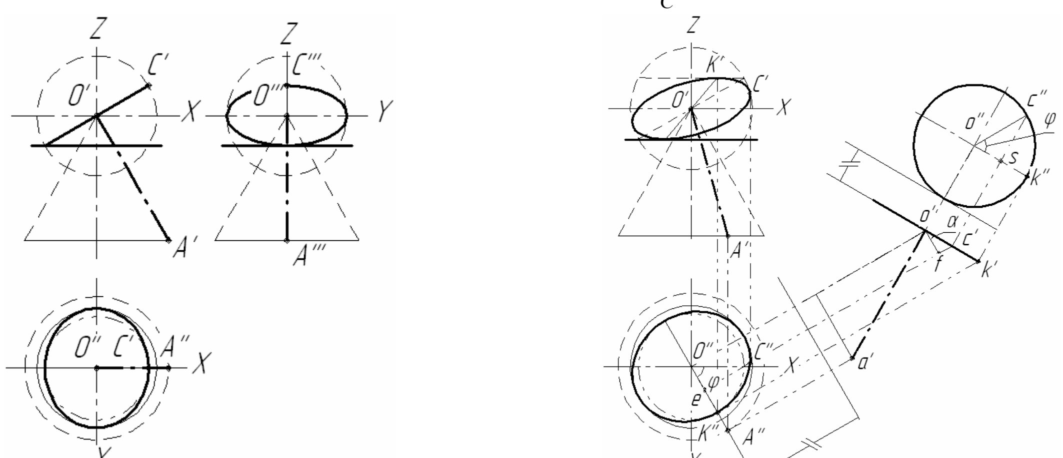
$$A_Y = z_A \sin\alpha \sin\varphi , \tag{2}$$

$$A_Z = z_A \cos\alpha , \tag{3}$$

where angle φ is used for determining the movement of solid's points that defines the angular position of crankshaft.

The simplified projections of the planes $XZ0$, $XY0$ and $YZ0$ for the motion process are considered on Figure 4a to define the movement of the point $C(x_C,0,0)$, where the radius of disk *1* is

$$R = x_C . \tag{4}$$



a) b)

Figure 4: The motion of a disc as a cone pendulum

Applying the auxiliary construction, the movement of the point $C(x_C,0,0)$ along the axis *XO* for the case of turning the axis 0_1z on angle φ relative to the axis *X0* on plane *XY0*, Figure 4b, can be calculated as below:

$$C_X = O''e \cdot \cos\varphi + eC'' \cdot \sin\varphi, \tag{5}$$

The following equations can be deduced for the subsequent reasoning:

$$O''e = o'f = o'c' \cdot \cos\alpha, \tag{6}$$

$$o'c' = o''s = o''c'' \cdot \cos\varphi = R \cdot \cos\varphi, \tag{7}$$

$$eC'' = sc'' = o''c'' \cdot \sin\varphi = R \cdot \sin\varphi. \tag{8}$$

Therefore

$$C_X = R \cdot (\cos^2\varphi \cdot \cos\alpha + \sin^2\varphi). \tag{9}$$

The movement of the point $C(x_C,0,0)$ depending on the angle φ can be calculated as below:
- along the axis *Y0*

$$C_Y = O''e \cdot \sin\varphi - eC'' \cdot \cos\varphi = 0.5 \cdot R \cdot (\cos\alpha - 1) \cdot \sin 2\varphi; \tag{10}$$

- along the axis *Z0*

$$C_Z = fc'' = o'c' \cdot \sin\alpha = R \cdot \cos\varphi \cdot \sin\alpha. \tag{11}$$

The motion of the point $B(x_B,y_B,0)$ определяется схожим образом как и перемещение точки $C(x_C,0,0)$, но с учетом того, что система координат $xyzO_1$ повернулось на угол β.

$$B_X = R \cdot [(\cos\beta \cdot (\cos^2(\varphi+\beta) \cdot \cos\alpha + \sin^2(\varphi+\beta)) - \\ -0.5 \cdot \sin\beta \cdot (\cos\alpha - 1) \cdot \sin 2(\varphi+\beta)], \tag{12}$$

$$B_Y = R \cdot [(\sin\beta \cdot (\cos^2(\varphi+\beta) \cdot \cos\alpha + \sin^2(\varphi+\beta)) + \\ +0.5 \cdot \cos\beta \cdot (\cos\alpha - 1) \cdot \sin 2(\varphi+\beta)], \tag{13}$$

$$B_Z = R \cdot \cos(\varphi+\beta) \cdot \sin\alpha, \tag{14}$$

where the radius is

$$R = \sqrt{{x_B}^2 + {y_B}^2}, \tag{15}$$

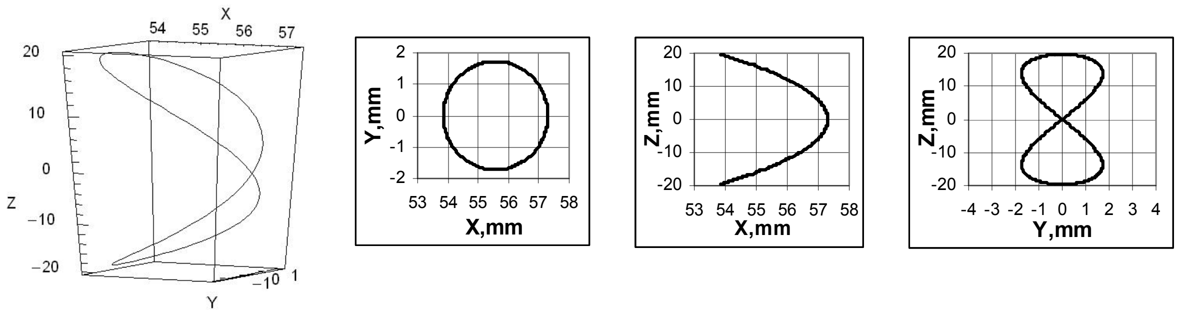
and β is the angle between the axis $x0_1$ and the vector of disposition $B(x_B,y_B,0)$ on the plane $xy0_1$. The angle β is also an angular position of the cylinder in the converting mechanism. It can be calculated by the positive coordinate x_B as follows

$$\beta = \arcsin\left(y_B \middle/ \sqrt{{x_B}^2 + {y_B}^2}\right). \tag{16}$$

From the system of equations above, it is possible to determine such kinematic characteristics as the trajectory, the velocity, the acceleration and the movement of point of the body determining the CP motion process.

Kinematics of developed axial-piston engine

The connection points of the connecting rods with the swing plate (see Figure 1) are located in the equatorial plane of the developed axial-piston engine. The distance *R* from the center of the EP is of 57.3 mm, the angle α between the prime axis and the crank axis is of 20°. The connection point trajectory of the first cylinder (β=0) is shown in Figure 6a; the trajectory projections on planes *XY0, XZ0* and *YZ0* are displayed on Figure 5b-d.



a) b) c) d)

Figure 5: The trajectory of the connection point of the first cylinder

The connection points of the connecting rods with the swing plate traces the "figure-of-eight" trajectory. The trajectory projection on the plane *XY0* is circular. The scale of the coordinate axis in the planes *XZ0* and *YZ0* is shown in different values to display better the "figure-of-eight" trajectory. Actually the trajectory is close to arc so the aberration along the axis *Y0* is less approximately in 11.34 times than the aberration along the axis *Z0*.

The maximum aberration on the axis *Y0* is $\Delta y_{max} = 1.728\ mm$ at angle $\varphi = \pi n/4$, where $n = 1,3,5...$; and y=0 at $\varphi = \pi k/2$ where $k = 0,1,2,$ ... The maximum aberration on the axis *X0* is $\Delta x_{max} = 2\cdot\Delta y_{max} = 3.456\ mm$ at $\varphi = \pi m/2$, where $m = 1,3,5...$ The motion of the point along the axis *Z0* is considered in detail in the next part of the paper.

Piston kinematics

The piston movement depending on the angle φ is determined as the alternate motion of a point along the cylinder axis linked through the connecting rod with a point on the equatorial plane. The piston kinematics with an adequate accuracy at a small angular aberration of a connecting rod from its original position can be observed as the motion of a point on the equatorial plane along the crank axis, i.e. along the axis *Z0*.

By analogy with the determination of movement of a point on EP along the axis *Z0*, namely

$$C_Z = R \cdot \cos\varphi \cdot \sin\alpha, \quad (17)$$

the piston movement can be defined as follows

$$s'(\varphi) = R \cdot \cos(\varphi - \beta) \cdot \sin\alpha = S/2 \cdot \cos(\varphi - \beta). \quad (18)$$

It is more practical to define a movement of the piston relative to the top dead center

$$s(\varphi) = \frac{S}{2} - s'(\varphi) = \frac{S}{2}(1 - \cos(\varphi - \beta)). \quad (19)$$

The velocity and the acceleration of the piston can be calculated as follows

$$v(\varphi) = \frac{d}{dt}\left(s(\varphi)\right) = \frac{d}{dt}\left(\frac{S}{2}(1 - \cos(\varphi - \beta))\right) = \frac{d}{dt}\left(\frac{S}{2}(1 - \cos(\omega \cdot t))\right) = \\ = \frac{S}{2}\omega\sin(\omega \cdot t) = \frac{S}{2}\omega\sin(\varphi - \beta), \quad (20)$$

$$a(\varphi) = \frac{d}{dt}\left(v(\varphi)\right) = \frac{S}{2}\omega^2\cos(\varphi - \beta), \quad (21)$$

where ω is the angular velocity of the crankshaft.

Actually the angular aberration of a connecting rod effects on the acceleration of the piston increasing insignificantly the movement value and an average velocity of the piston. The connecting rod length of the designed APE is $L=60$ *mm*, therefore the greatest angular aberration of a connecting rod can be calculated as

$$\psi = \arcsin\frac{\Delta x_{\max}}{L} = \arcsin\frac{3.456}{60} = 3.3^{\circ}. \qquad (22)$$

For reference, the maximal rod deflection in the Yamaha FZ6-S engine is of 14^{o}.
As a result, the maximum inaccuracy of the Eq. (15) for the first cylinder of the developed APE is defined at $\varphi = 90^{o}$ as

$$\Delta = \frac{L(1-\cos\psi)}{S/2}\cdot 100\% = \frac{60(1-0.99834)}{39.2/2}\cdot 100\% = 0.508\%. \qquad (23)$$

The piston kinematics, namely the movement, the velocity and the acceleration, is close to a harmonic relation. The piston stands an equal period nearly the dead-center position each that increases the fuel injection time. On the other hand, this reduces the time to scavenge the exhaust gases under the same conditions when the piston stands nearly the bottom dead point. For reference, the piston movement for 10^{o} after the top dead center by engine speed of *10000 rpm* is:
- *0.4196 mm* for the inclined engine,
- *0.2978 mm* for the developed APE; the movement should be of *0.3880 mm* for the piston stroke of *44.5 mm* as for inclined engine.

DYNAMICS

The main task of dynamics is to study the movement of a solid in relation to the operating forces. The most interesting in the dynamics of the CP converting mechanism is the dynamic compensation together with the compensation conditions as applied to engines. The dynamic compensation is close related to the inertial forces from the moving masses (2). They load extra the crankshaft bearing. The loading increases by the growth of the crankshaft revolution frequency. The simplified dynamic model of the CP converting mechanism includes the center with the mass m_S and n points with the mass m_j. These points are equidistant from the center of the equatorial plane. The dynamic model is presented on Figure 6a. The masses of the piston assembly, the connecting rod and the part of the swing plate are compiled in a point, which connects the rod with the swing plate.
Considering the CP converting mechanism as dynamic model, it can be written:

$$M_S = m_S + n\cdot m_j, \;\; n\cdot m_j R^2 = I_k, \qquad (24)$$

where M_S is the total mass of the dynamic model, I_k is the moment inertia of the dynamic model relative to the prime axis k.

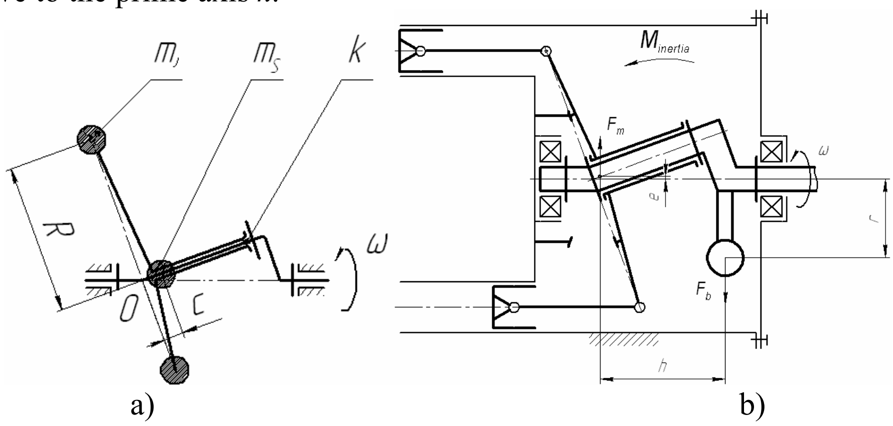


Figure 6: Dynamic models: *O* – center of mass.

Therefore

$$m_j = I_k / nR^2 \, , \; m_S = M_S - n \cdot m_j \, . \tag{25}$$

A point j of the dynamic model moves with the acceleration. The projections of acceleration on the axis *X0, Y0* and *Z0* at the certain *i*-moment of time can be written as

$$a_{xji} = 2R\omega^2 (1 - \cos\alpha) \cdot [\cos 2(\varphi_i + \beta_j) \cdot \cos\beta + \sin 2(\varphi_i + \beta_j) \cdot \sin\beta] \, , \tag{26}$$

$$a_{yji} = 2R\omega^2 (1 - \cos\alpha) \cdot [\cos 2(\varphi_i + \beta_j) \cdot \sin\beta - \sin 2(\varphi_i + \beta_j) \cdot \cos\beta] \, , \tag{27}$$

$$a_{zji} = -R\omega^2 \sin\alpha \cdot \cos(\varphi_i + \beta_j) \, . \tag{28}$$

The inertial force moments of *j*-point at *i*-moment of time can be calculated as

$$M_{xji} = m_j (y_{ji} a_{zji} - z_{ji} a_{yji}) \, , \tag{29}$$

$$M_{yji} = m_j (z_{ji} a_{xji} - x_{ji} a_{zji}) \, , \tag{30}$$

$$M_{zji} = m_j (x_{ji} a_{yji} - y_{ji} a_{xji}) \, , \tag{31}$$

where x_{ji}, y_{ji}, z_{ji} are the coordinates relative to the axis *X0, Y0* and *Z0*:

$$x_{ji} = R \cdot [(\cos\beta_j \cdot (\cos^2(\varphi_i + \beta_j) \cdot \cos\alpha + \sin^2(\varphi_i + \beta_j)) - \\ -0.5 \cdot \sin\beta_j \cdot (\cos\alpha - 1) \cdot \sin 2(\varphi_i + \beta_j)] \, , \tag{32}$$

$$y_{ji} = R \cdot [(\sin\beta_j \cdot (\cos^2(\varphi_i + \beta_j) \cdot \cos\alpha + \sin^2(\varphi_i + \beta_j)) + \\ +0.5 \cdot \cos\beta_j \cdot (\cos\alpha - 1) \cdot \sin 2(\varphi_i + \beta_j)] \, , \tag{33}$$

$$z_{ji} = R \cdot \sin\alpha \cdot \cos(\varphi_i + \beta_j) \, . \tag{34}$$

The summarized moments can be defined as

$$M_{xi} = \sum_{j=1}^{n} M_{xji} \, , \; M_{yi} = \sum_{j=1}^{n} M_{yji} \, , \; M_{zi} = \sum_{j=1}^{n} M_{zji} \, . \tag{35}$$

The magnitude of the inertial force moment is

$$M_{inertia} = M_i = \sqrt{M_{xi}{}^2 + M_{yi}{}^2 + M_{zi}{}^2} \; . \tag{36}$$

It is important to mark that the magnitude of the inertial force moment is constant and circulates in the crank plane of the converting mechanism.

The inertial force moment can be balanced by the rotation moment of the center mass and the balancing weight. The balancing weight is located on the crank plane. The descriptive compensation is presented in Figure 7b. The dynamic compensation together with the compensation conditions in relation to engines with cone pendulum mechanism is as follows.

- Join points of the connecting rods with swing plate should be placed on the equatorial plane;
- The moment of inertia of the swing plate relative to its axis in the equatorial plane should be constant that is possible, for example, at odd amount of pistons (3,5,7,…);
- The inertial force moment is balanced by selecting such dynamic compensation parameter as the mass of the balancing weight m_b, the radius r, the arm h:

$$M_{inertia} = (m_S \cdot e + m_b \cdot r) \cdot h \cdot \omega^2 / 2 \, . \tag{37}$$

It is possible to develop the ω^2 from the equations

$$M_{inertia} = M_i = \sqrt{M_{xi}{}^2 + M_{yi}{}^2 + M_{zi}{}^2} \; , \tag{38}$$

$$M_{inertia} = (m_S \cdot e + m_b \cdot r) \cdot h \cdot \omega^2 / 2 \; . \tag{39}$$

Hence, the moment from the inertial force is balanced by installation of a counterweight on the prolongation of the crank plan irrespective of an angular velocity of the shaft.
The main specification of the developed APE, which is required to define the inertial force moment and the dynamic compensation parameters, is presented in Table 2.

Table 2: Technical parameters of the developed axial-piston engine

Quantity of cylinder	3
Distance from the center of the EP, *R*	0.0573 m
Angular velocity, ω	1256.6 rps
Angular position of the first cylinder, β_1	0°
Angular position of second cylinder, β_2	120°
Angular position of third cylinder, β_3	240°
Total mass, M_S	3.08 kg
Moment inertia concerning the prime axis, I_k	0.00899 kg·m^2
Distance between the center of mechanism *0* and the center of mass, *c*	0.007 m
Radius, *r*	0.045 m
Shoulder, *h*	0.0946 m
Radius of the center of the EP, *e*	0.0024

Some parameters from the Table 2 have been defined by means of CAD systems as Solid works. The mass of the point of join and the center of mass is follows

$$m = m_1 = m_2 = m_3 = I_k / nR^2 = 0.00899/(3 \cdot 0.0573^2) = 0.913 \text{ kg}, \quad (40)$$

$$m_S = M_S - n \cdot m_j = 3.08 - 3 \cdot 0.913 = 0.341. \quad (41)$$

The inertial force moments of the first point at *i*-moment of time are

$$M_{x1i} = m(y_{1i}a_{z1i} - z_{1j}a_{y1i}), M_{y1i} = m_1(z_{1i}a_{x1i} - x_{1i}a_{z1i}), M_{z1i} = m_i(x_{1i}a_{y1i} - y_{1i}a_{x1i}). \quad (42)$$

The diagram of the inertial force moments of the first point is shown on Figure 7.
The summarized moments relative to each of axes at the *i*-moment of time are

$$M_{xi} = M_{x1i} + M_{x2i} + M_{x3i}, M_{yi} = M_{y1i} + M_{y2i} + M_{y3i}, M_{zi} = M_{z1i} + M_{z2i} + M_{z3i}. \quad (43)$$

The magnitude of the inertial force moment is

$$M_{inertia} = M_i = \sqrt{M_{xi}^2 + M_{yi}^2 + M_{zi}^2} = 2900.5 \text{ Nm}. \quad (44)$$

The diagram of the summarized moments and the magnitude of the inertial force moment is shown on Figure 8.

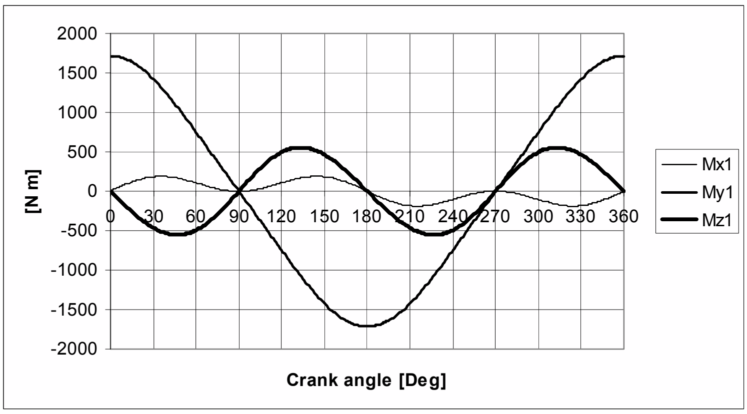


Figure 7: The diagram of the inertial force moments

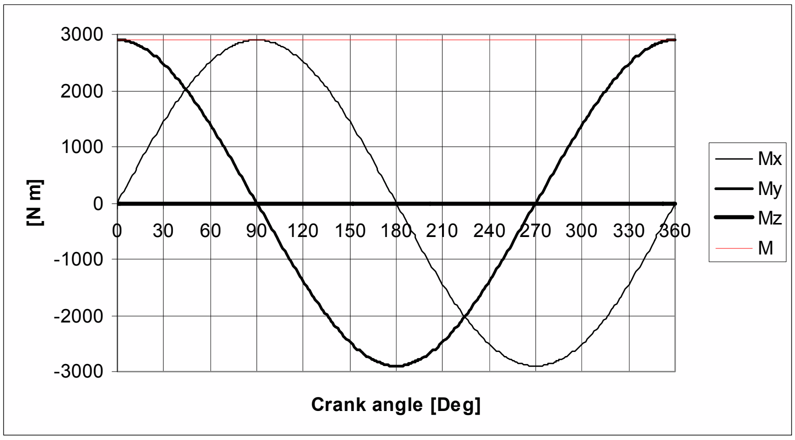


Figure 8: The diagram of the summarized moments and the magnitude of the inertial force moment

$$m_b = \frac{1}{r}\left(\frac{2 \cdot M_{inertia}}{h \cdot \omega^2} - m_S \cdot e\right) = \frac{1}{0.045}\left(\frac{2 \cdot 2900.5}{0.0946 \cdot 1256.6^2} - 0.341 \cdot 0.0024\right) = 0.844 \text{ kg.} \tag{45}$$

The mass of balancing weight can be increased to unload the crank bearing from inertial forces caused by the rotational motion of the crank masses.

CONCLUSIONS

1) The developed axial-piston engine is more compact in 1.5 times in comparison with an analogous inclined engine.
2) The crankshaft of 3-cylinder APE has only two bearings. It simplifies the construction and allows to install the frictionless bearings and to reach the small relative velocities in spherical joints that improves the mechanical efficiency.
3) The piston kinematics in APE is close to the harmonic motion. The piston is positioned on the top dead center longer than in the traditional engine. This increases time for the fuel injection, contributes to the free gas exhaust, reduces the engine noise and stiffness, all things being equal.
4) The small deflection of the connecting rod from its primary axis reduces the lateral loading on the cylinder mirror and piston skirt.
5) A piston of APE has the regular cylindrical form with the equal distribution of the material. The piston diameter wears uniformly during the operation. Both features ease the requirements to the piston design.
6) The crank mechanism is dynamically balanced via a counterweight on the extension of the crank web. No additional shafts are required.

All features mentioned show that the developed concept of an axial-piston engine is a real alternative to traditional engines especially for machines demanding small dimension and mass. The good dynamic balancing of APE allows its forcing on the crankshaft frequency.

REFERENCES

(1) Yamaha FZ6-S(S) Service manual 2003 Yamaha Motor Co., Ltd. First edition, 2003.
(2) Yamanin, A.I., Zahrov, A.V. Dynamics of reciprocating motors, Moscow, Mechanical Engineering Publishing, 2003.
(3) http://carrilloind.com/Portals/1/Documents/carrillo2008.pdf%5B1%5D.pdf
(4) http://moto-pro.ru/content/view/9/33/
(5) http://en.wikipedia.org/wiki/Precession