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## **THE ROLLOVER MODELLING OF A LIGHT OFF-ROAD VEHICLE WITH CONSIDERATION OF SOIL EFFECT**

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KEYWORDS – Rollover, Off-road vehicle, Suspension, Tire, Soil

ABSTRACT - The objective of this paper is to develop a new model of un-tripped rollover in off-road vehicles which is not very well understood yet. Furthermore, a new approach is presented to show the rollover phenomenon in this kind of vehicle. In addition, in contrast with other performed works, the rollover is assumed in a transient mode between tripped and un-tripped rollover where the effect of soil compaction is considered. Taking into account that the soil under load is an easily deformable pattern, we can define two conditions for it; sinkage and compaction. Consequently, consider the changing of the loads on wheels, the tires will sinkage through the soil and as a result there would be the compaction of soil behind the outside tires which may cause to rollover incident. A Mini Baja, which is a light off-road vehicle and is categorized in buggies, was used for all tests and simulation.

### 1. INTRODUCTION

In order to describe the behaviour of vehicle at the threshold of rolling over in transient state, we first consider the vehicle in steady state cornering. Lateral acceleration relation at the threshold of rollover in steady state cornering will be obtained. Then the behavior of the vehicle in transient situation will be obtained by introducing some slight changes in the steady state equation.

### 2. LATERAL ACCELERATION OF VEHICLE IN STEADY STATE CORNERING

To obtain lateral acceleration at rollover threshold in steady state cornering, in addition to mass, track width, height of center of gravity and roll center of vehicle, some other factors like, lateral movement of vehicle center of gravity due to rolling, lateral movement of vehicle due to kinematics of suspension system and elevation in the height of center of gravity of vehicle due to jacking forces have been taken into account. Other factors such as nonlinearity and lateral compliance of suspension system are not considered because their variation from one vehicle to another is usually pronounced and could be introduced if one special vehicle is to be analyzed. Note that some other factors such as damping will be introduced in analysis of dynamic state. It should be noted that in the following stages, roll center is assumed to be the same for front and rear parts of vehicle. Also it is very important that following relations are written for the state in which vehicle is at the threshold of rollover.

#### 2.1 Lateral movement of center of gravity due to rolling

One simple model of roll over of vehicles was suggested by Garrot et al., [1] which predicts that roll over occurs when lateral acceleration during cornering reaches “ $g. S.S.F$ ” and defines:

$$S.S.F = \frac{t_w}{2h_0} \quad (1)$$

$t_w$  is track width and  $h_0$  is the initial height of center of gravity. This model supposes the vehicle as a rigid body. This simple model overestimates the lateral acceleration of vehicle at the rollover threshold [2]. To introduce the effect of roll angle, consider Fig.1.  $\Phi$  is roll angle and could be obtained by equation (2).

$$\Phi = \frac{M_s \cdot h_{roll} \cdot a_y}{k_\Phi} \quad (2)$$

The numerator expresses the exerted moment about roll center and the denominator denotes the total rotational stiffness of vehicle (including effect of tire compliance).

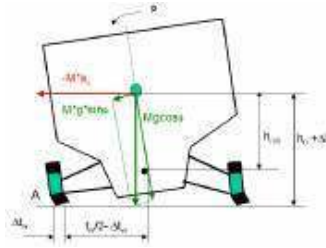


Figure 1: Vehicle Model [2].

## 2.2 Change in track width during steady cornering

Change in track width of the vehicle is caused mainly by two factors, first one lateral compliance of tires and second, kinematics of suspension. Here it is assumed that the behaviour of tires is linear. So reduction in track width due to tire compliance could be obtained by equation (3).

$$\Delta t'_w = \frac{M \cdot a_y}{k_{It}} \quad (3)$$

Where  $k_{It}$  and  $M$  denotes lateral stiffness of tires and total mass respectively. In order to take the second factor into account, one should consider the roll center of tires with respect to body. So tire movement with respect to the body is approximately perpendicular to the line "Ac"; approximately  $\Delta z = \Phi \cdot t_w / 2$ . So  $\Delta t''_w$ , that is increase in track width due to suspension kinematics could be expressed as equation (4).

$$\Delta t''_w = \Delta z \cdot \tan \gamma = \frac{M_s \cdot h_{rollc} \cdot h_{roll} \cdot a_y}{k_\Phi} \quad (4)$$

Since  $\Delta t'_w$  and  $\Delta t''_w$  are reduction and increment in track width respectively, total decrease in track width would be:

$$\Delta t_w = M_s . a_y \left( \frac{h_{rollc} . h_{roll}}{k_{\Phi}} - \frac{1}{k_{lt}} \right) \quad (5)$$

### 2.3 Effect of jacking forces

Gillespie [4] showed that forces transmitted between wheel and body of the vehicle are equivalent to a force that acts through the line connecting the center of tires contact patch to roll center. The vertical component of this force tends to elevate center of gravity of body and is called “jacking force”. At the threshold of rollover, the lateral inertia force of vehicle must be counterbalanced by the horizontal component of “ $F$ ”, since inside wheels provide no vertical or lateral force.

$$F_v = F_l . \tan \gamma = \frac{2M_s h_{rollc} a_y}{t_w} \quad (6)$$

Where  $F_v$  and  $F_l$  denote components of  $F$  in vertical and lateral directions respectively. Jacking force results in change in height of center of gravity of the body. This change in height is:

$$\Delta h = \frac{F_v}{k_{vt}} = \frac{2M_s h_{rollc} a_y}{k_{vt} t_w} \quad (7)$$

Where  $k_{vt}$  denotes total vertical stiffness of vehicle suspension. Substituting this amount of acceleration in equation (6) yields:

$$\Delta h = 0.8(h_{rollc} / h_0)(M_s g / k_{vt}) \quad (8)$$

### 2.4 Lateral acceleration at the rollover threshold

Lateral acceleration at the rollover threshold is obtained by taking moment about outside tires. Referring to equation of equilibrium, the following relation can be written:

$$\sum T_A = M a_y (h_0 + \Delta h) - Mg \cos \Phi (t_w / 2 - \Delta t_w) + Mg \sin \Phi \cos \Phi h_0 = 0 \quad (9)$$

Letting  $\sin \Phi = \Phi$  and  $\cos \Phi = 1$  for small roll angle, equation (9) can be written down as:

$$a_y = \frac{g . SSF}{[1 + \Delta h / h_0 + M_s g h_{roll} (1 - h_{rollc} / h_0) / k_{\Phi} + Mg / (k_{lt} h_0)]} \quad (10)$$

In the following, the relation for lateral acceleration at the rollover threshold in steady state cornering is going to be developed into the transient state to take account of effect of soil.

## 3. THE ROLLOVER THRESHOLD AT TRANSIENT MODE

In order to develop equation (10) from steady state to transient state in which lateral inertia force is not constant, a simplified model of vehicle is used (Fig.2). Equation of motion for this model is stated in equation (11). Thereby the effect of varying tangential force exerted on tires by soil is introduced to the equation (10).

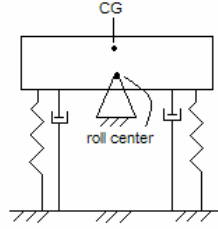


Figure 2: Vehicle model

$$I_s \frac{d^2\Phi}{dt^2} + C_\Phi \frac{d\Phi}{dt} + k_\Phi \Phi = T(t) \quad (11)$$

Where  $I_s$  denotes the inertia of moment of the sprung mass about center of gravity, and  $C_\Phi$  and  $k_\Phi$  denotes total roll damping and total roll stiffness of the vehicle and  $T(t)$  is the moment exerted about the roll center of the body.  $T(t)$  is generally a function of time which for a especial case of steady-state cornering it is constant. For example for a vehicle which passes from an ideally smooth road to a rough one (during cornering),  $T(t)$  exhibit a step function as depicted in Fig.3.

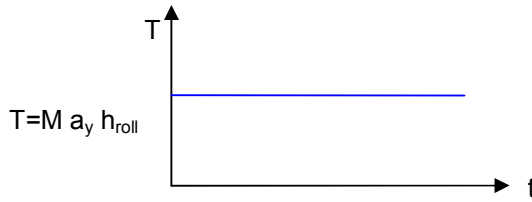


Figure 3: Exerted moment on model during transition from smooth to rough road

Assuming applied moment as a step function, solving of equation (11) for  $\Phi$  is a curve oscillating around  $\Phi_s$  (steady-state roll angle which is obtained by substitution of equation (10) into equation (2)). Therefore,  $\Phi_{max}$  would be:

$$\Phi_{max} = \Phi_s (1 + \Delta_{os}) \quad (12)$$

Where,  $\Delta_{os}$  could be obtained by equations (13) and (14):

$$\Delta_{os} = e^{\left(\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}\right)} \quad (13)$$

$$\zeta = \frac{C_\Phi}{2\sqrt{I_s k_\Phi}} \quad (14)$$

Development of the steady-state relation (equation (10)) to a relation that also consider the effects of step moment function (not constant moment) is done by multiplying  $h_{roll}$  in equation (10) by  $(1+\Delta_{os})$ . It could be conformed by deriving equation (10) considering  $\Phi_{max}$  rather than  $\Phi$ .  $T(t)$  as a step function of time introduces an example of developing equation (10) to non-steady state occasions.

### 3.2 Effect of soil on lateral acceleration at the rollover threshold

The process of considering the soil impact is identical to the process introduced in the step function  $T(t)$  in previous section. Once again equation (11) must be solved by substituting appropriate  $T(t)$ .

## 4. DIRECT STUDY OF ROLLOVER

During maneuvering or turning the vehicle around a curve, there exist the effect of tire sinkage through the soil and consequently the compaction of soil behind the outside tire. Therefore, in this article, we consider this situation similar to a tripped rollover and we just consider the effect of soil compaction, which leads to rollover of an off-road vehicle (Fig. 4).

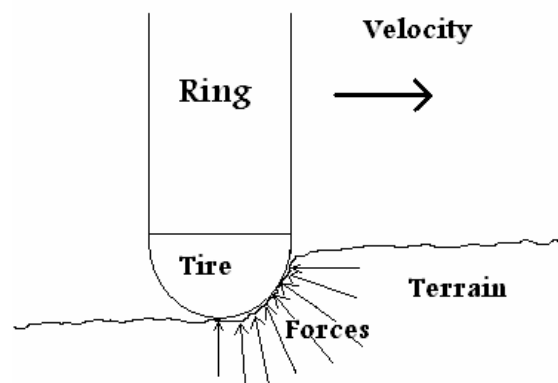


Figure 4: Graphic simulation of tire while turning on a given terrain.

### 4.1 Study of vehicle's behaviour while cornering on soil

Different behaviors of the vehicle in turns depend on the type of suspension and the road surface as well as the dynamic characteristics and the Understeer and Oversteer gradient. If the wheels were not able to obtain the cohesion and friction forces on the road surface, the vehicle would slip and would deviate.

Soil is a three phase's material and can be compacted easily. It has low shear resistant and the layers of soil can easily slip on their selves. Also the relation between shear resistant and axial resistant of that is linear. This means that by increasing the normal force on the soil, the shear resistant will increase and vice versa. In addition, the reaction of soil is directly depending on the pressure of contact area of the part on the soil. This means that in investigating the behaviors of soil and tire, the important parameter is pressure in contact patch, and the load on the tire is not important by itself [5].

For an off-road vehicle in turning the vehicle has lateral slip because of shearing in the soil layers. In other hand, there is lateral load transition because of lateral acceleration.

Consequently, by increasing the load on tire, shearing resistant of soil will be increased and the lateral slip will decreased. Accordingly, behavior of an off-road vehicle in maneuvering is depending on shear strength and sinkage in contact patch. In order to obtain the desired equations, the two factors can be investigated separately by assuming certain moisture for soil, negligible tire deformation, semicircular profile for tire and controlled soil conditions.

#### 4.2 Computation and converge the tests

Inasmuch as the tire profile has been assumed semicircular, as a result it will not change during sinkage. To some extent, instead of working with the side force and vertical load on tire that has much error, the computations are based on shear and normal stresses at the circular profile. Then, by converging the calculated side and horizontal forces in different conditions, the main expression will be emerged. Performed test toward this work are:

1. Coulomb envelope and the soil shear resistant determination.
2. Sinkage and tire pressure (pressure on contact patch) determination.
3. Pressure on contact patch and load on tire.

### 5. RESULTS AND DISCUSSION

The plate sinkage (PST) and confined compression test (CCT) to find shear resistant and sinkage respectively have been done in a way that normal stresses and actuated force be correspondence to the stress in the contact area of front and rear tire. The weight distribution of the tested vehicle was 45% in front and 55% in rear and according to the total mass of 400 Kg for the vehicle, the load on front tires was 180 Kg and the load on rear tire was 220 Kg. This vehicle prevailed R 7X10 tires with a contact patch of 428 cm<sup>2</sup> for each tire in static conditions. The parameters of coulomb equation (c and  $\phi$  in equation (15)) were elicited from the linearizing points in Excel (Fig. 5).

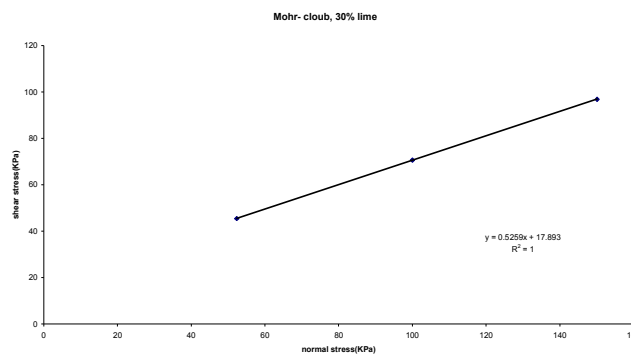


Figure 5: Mohr-Coulomb Diagram

The equation of Mohr-Coulomb after perform PST with a 70.88 cm<sup>2</sup> Plate and a 883 N applied force on a soil with 30% lime and 21% moisture was gained.

$$\tau = c + \sigma \tan \phi \quad (15)$$

$$Y=0.5289X+17.893 \quad (16)$$

$$A=70.88 \text{ cm}^2 \quad (17)$$

$$\sigma = \frac{883N}{70.88\text{cm}^2} = 124.5\text{KPa} \quad (18)$$

$$C=17.893 \text{ KPa} \quad (19)$$

$$\phi = \tan^{-1} 0.5289 = 27.87^\circ \quad (20)$$

$$\tau = 17.893 + 0.5289\sigma \quad (21)$$

While cornering, the operated lateral force on tires will change because of changing in lateral cross section area of tire. Furthermore, in the corners, because of lateral load transfer, the load on the tires and so the normal stress will be changed. With changing the normal stress, the shear resistance of the soil will change accordingly and the sinkage will be changed too. Consequently, the lateral force that is function of shear resistant and lateral tire area will be changed. Hence, the lateral force is a function of sinkage and normal stress in contact area.

The experiments related to soil sinkage with consideration of normal stress changing were done in conditions of the regular weight distribution in full stability and the rollover threshold with maximum lateral load transfer, in order to find the soil sinkage (table 1).

Normal stress=71 kPa		normal stress=100 kPa	
$\sigma$ (kPa)	$z$ (mm)	$\sigma$ (kPa)	$z$ (mm)
32.27	25	34.28	25
39.33	34	42.35	36
44.37	41	50.01	46
48.4	45	56.47	52
51.43	48	60.5	54
53.24	49	64.13	59
55.06	51	65.74	63.5
57.68	52	66.55	68
58.48	53	67.56	75

Table 1: CCT test

According to obtained diagrams, normal stress-sinkage relation in two boundary conditions of full stability and rollover threshold acquired (figures 6 and 7).

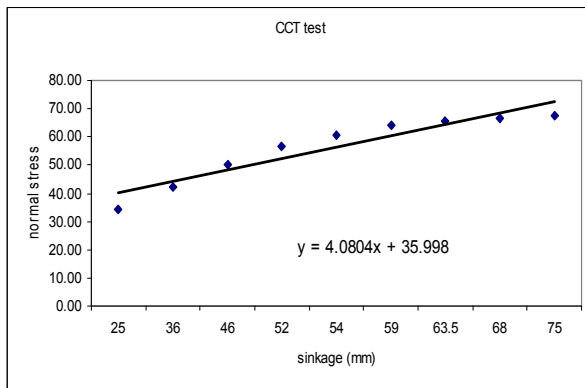


Figure 6: CCT test with 70 kPa normal stress

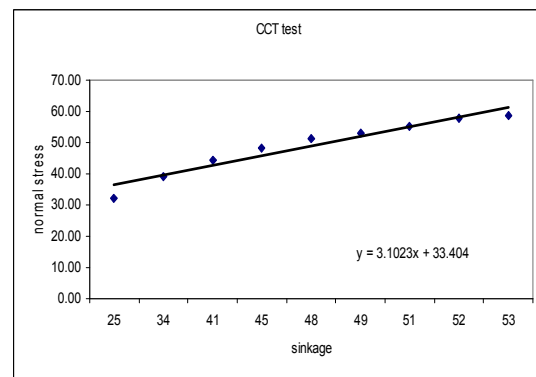


Figure 7: CCT test with 100 kPa normal stress

$$\sigma_s = 33.404 + 3.1023z \quad (22)$$

$$\sigma_r = 35.998 + 4.08z \quad (23)$$

In a way, according to coulomb relation (equation 21) and equations 22 and 23 it gained:

$$\tau_s = 51.071 + 1.642z \quad (24)$$

$$\tau_r = 36.932 + 2.157z \quad (25)$$

In result of lateral load transfer and soil sinkage and considering the table 1, side section will change in this way:

$$x = r^2 - (r - z)^2 = (z(2r - z))^{0.5} \quad (26)$$

$$\theta = \tan^{-1}\left(\frac{\sqrt{r^2 - (r - z)^2}}{r - z}\right) \quad (27)$$

According to above equations, the contact area will calculated in the following form:

$$A_l = \theta\pi r^2 - 0.5(r - z)(z(2r - z))^{0.5} \quad (28)$$

Considering the equations 24, 25 and 28 and

$$F_l = \int_0^z \tau dA \quad (29)$$

The tire-terrain lateral relation will be obtained:

$$F_l = (51.071 + 1.642z) * \left(\tan^{-1}\left(\frac{\sqrt{r^2 - (r - z)^2}}{r - z}\right)\right) * (\pi r^2 - 0.5(r - z)(z(2r - z)))^{0.5} \quad (30)$$

## 6. CONCLUSIONS

The main theory of the paper has been discussed. However, this theory can be extended by performing some other tests, which needs more facilities. Nonetheless, this research will leads to more accurate understanding of interaction between off-road and terrain.

Furthermore, the above experimental equations to calculate the forces and dynamic and static behaviors of the vehicle will be helpful to more clear study of rollover of vehicle. Moreover, the extended model is the best to describe the pressure-sinkage and shear stress-displacement relationships along the contact patch between tyre and terrain, which describe the roll over behavior of vehicle more accurate than previous models. Finally, using the result of this paper in designing the off-road vehicles will increase the rollover threshold and consequently the safety of vehicle.

## 7. REFERENCES

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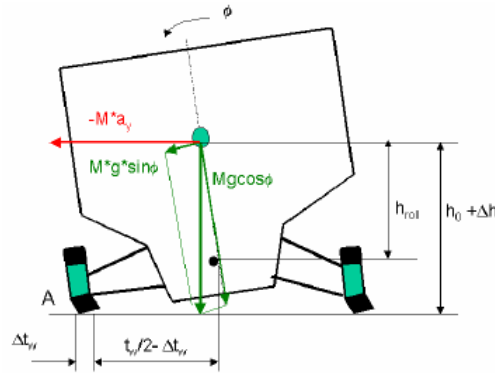


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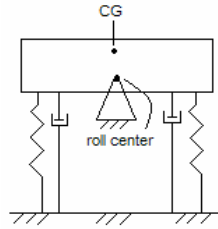


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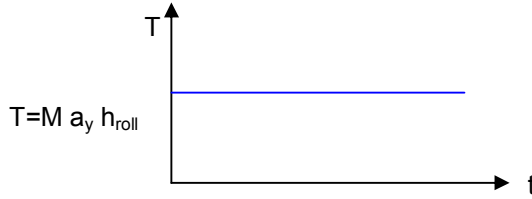


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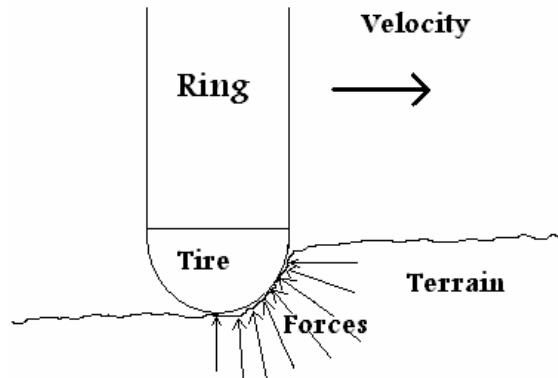


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1. Coulomb envelope and the soil shear resistant determination.
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### 5. RESULTS AND DISCUSSION

The plate sinkage (PST) and confined compression test (CCT) to find shear resistant and sinkage respectively have been done in a way that normal stresses and actuated force be correspondence to the stress in the contact area of front and rear tire. The weight distribution of the tested vehicle was 45% in front and 55% in rear and according to the total mass of 400 Kg for the vehicle, the load on front tires was 180 Kg and the load on rear tire was 220 Kg. This vehicle prevailed R 7X10 tires with a contact patch of 428 cm<sup>2</sup> for each tire in static conditions. The parameters of coulomb equation ( $c$  and  $\phi$  in equation (15)) were elicited from the linearizing points in Excel (Fig. 5).

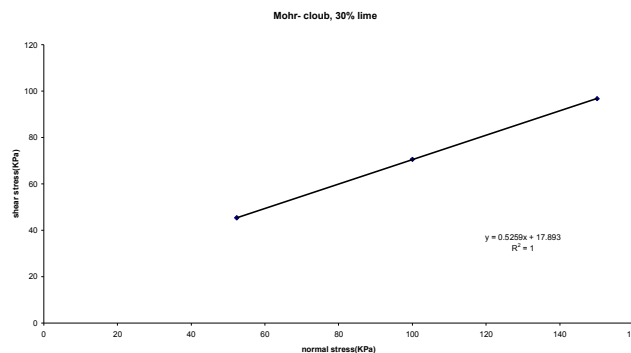


Figure 5: Mohr-Coulomb Diagram

The equation of Mohr-Coulomb after perform PST with a 70.88 cm<sup>2</sup> Plate and a 883 N applied force on a soil with 30% lime and 21% moisture was gained.

$$\tau = c + \sigma \tan \phi \quad (15)$$

$$Y=0.5289X+17.893 \quad (16)$$

$$A=70.88 \text{ cm}^2 \quad (17)$$

$$\sigma = \frac{883N}{70.88\text{cm}^2} = 124.5\text{KPa} \quad (18)$$

$$C=17.893 \text{ KPa} \quad (19)$$

$$\varphi = \tan^{-1} 0.5289 = 27.87^\circ \quad (20)$$

$$\tau = 17.893 + 0.5289\sigma \quad (21)$$

While cornering, the operated lateral force on tires will change because of changing in lateral cross section area of tire. Furthermore, in the corners, because of lateral load transfer, the load on the tires and so the normal stress will be changed. With changing the normal stress, the shear resistance of the soil will change accordingly and the sinkage will be changed too. Consequently, the lateral force that is function of shear resistant and lateral tire area will be changed. Hence, the lateral force is a function of sinkage and normal stress in contact area.

The experiments related to soil sinkage with consideration of normal stress changing were done in conditions of the regular weight distribution in full stability and the rollover threshold with maximum lateral load transfer, in order to find the soil sinkage (table 1).

Normal stress=71 kPa		normal stress=100 kPa	
$\sigma$ (kPa)	z (mm)	$\sigma$ (kPa)	z(mm)
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According to obtained diagrams, normal stress-sinkage relation in two boundary conditions of full stability and rollover threshold acquired (figures 6 and 7).

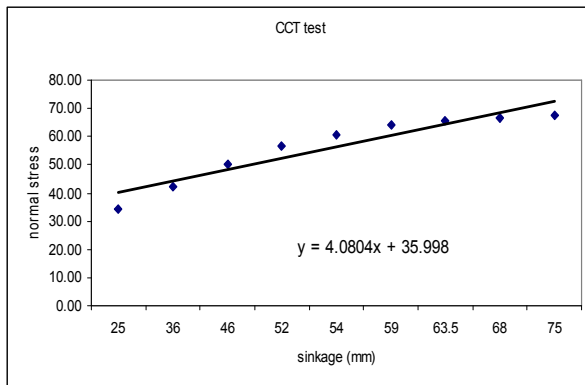


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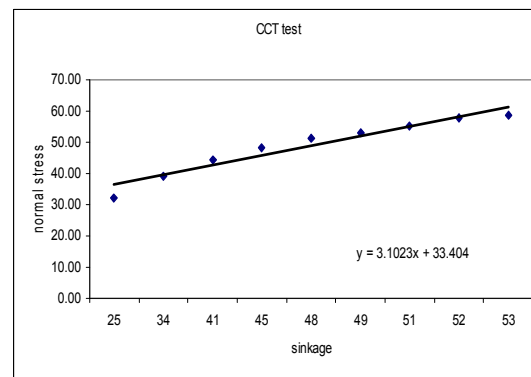


Figure 7: CCT test with 100 kPa normal stress

$$\sigma_s = 33.404 + 3.1023z \quad (22)$$

$$\sigma_r = 35.998 + 4.08z \quad (23)$$

In a way, according to coulomb relation (equation 21) and equations 22 and 23 it gained:

$$\tau_s = 51.071 + 1.642z \quad (24)$$

$$\tau_r = 36.932 + 2.157z \quad (25)$$

In result of lateral load transfer and soil sinkage and considering the table 1, side section will change in this way:

$$x = r^2 - (r - z)^2 = (z(2r - z))^{0.5} \quad (26)$$

$$\theta = \tan^{-1}\left(\frac{\sqrt{r^2 - (r - z)^2}}{r - z}\right) \quad (27)$$

According to above equations, the contact area will calculated in the following form:

$$A_l = \theta\pi r^2 - 0.5(r - z)(z(2r - z))^{0.5} \quad (28)$$

Considering the equations 24, 25 and 28 and

$$F_l = \int_0^z \tau dA \quad (29)$$

The tire-terrain lateral relation will be obtained:

$$F_l = (51.071 + 1.642z) * \left(\tan^{-1}\left(\frac{\sqrt{r^2 - (r - z)^2}}{r - z}\right)\right) * (\pi r^2 - 0.5(r - z)(z(2r - z)))^{0.5} \quad (30)$$

## 6. CONCLUSIONS

The main theory of the paper has been discussed. However, this theory can be extended by performing some other tests, which needs more facilities. Nonetheless, this research will leads to more accurate understanding of interaction between off-road and terrain. Furthermore, the above experimental equations to calculate the forces and dynamic and static behaviors of the vehicle will be helpful to more clear study of rollover of vehicle. Moreover, the extended model is the best to describe the pressure-sinkage and shear stress-displacement relationships along the contact patch between tyre and terrain, which describe the roll over behavior of vehicle more accurate than previous models. Finally, using the result of this paper in designing the off-road vehicles will increase the rollover threshold and consequently the safety of vehicle.

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## **THE ROLLOVER MODELLING OF A LIGHT OFF-ROAD VEHICLE WITH CONSIDERATION OF SOIL EFFECT**

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Isfahan University of Technology, Iran

KEYWORDS – Rollover, Off-road vehicle, Suspension, Tire, Soil

ABSTRACT - The objective of this paper is to develop a new model of un-tripped rollover in off-road vehicles which is not very well understood yet. Furthermore, a new approach is presented to show the rollover phenomenon in this kind of vehicle. In addition, in contrast with other performed works, the rollover is assumed in a transient mode between tripped and un-tripped rollover where the effect of soil compaction is considered. Taking into account that the soil under load is an easily deformable pattern, we can define two conditions for it; sinkage and compaction. Consequently, consider the changing of the loads on wheels, the tires will sinkage through the soil and as a result there would be the compaction of soil behind the outside tires which may cause to rollover incident. A Mini Baja, which is a light off-road vehicle and is categorized in buggies, was used for all tests and simulation.

### 1. INTRODUCTION

In order to describe the behaviour of vehicle at the threshold of rolling over in transient state, we first consider the vehicle in steady state cornering. Lateral acceleration relation at the threshold of rollover in steady state cornering will be obtained. Then the behavior of the vehicle in transient situation will be obtained by introducing some slight changes in the steady state equation.

### 2. LATERAL ACCELERATION OF VEHICLE IN STEADY STATE CORNERING

To obtain lateral acceleration at rollover threshold in steady state cornering, in addition to mass, track width, height of center of gravity and roll center of vehicle, some other factors like, lateral movement of vehicle center of gravity due to rolling, lateral movement of vehicle due to kinematics of suspension system and elevation in the height of center of gravity of vehicle due to jacking forces have been taken into account. Other factors such as nonlinearity and lateral compliance of suspension system are not considered because their variation from one vehicle to another is usually pronounced and could be introduced if one special vehicle is to be analyzed. Note that some other factors such as damping will be introduced in analysis of dynamic state. It should be noted that in the following stages, roll center is assumed to be the same for front and rear parts of vehicle. Also it is very important that following relations are written for the state in which vehicle is at the threshold of rollover.

#### 2.1 Lateral movement of center of gravity due to rolling

One simple model of roll over of vehicles was suggested by Garrot et al., [1] which predicts that roll over occurs when lateral acceleration during cornering reaches “*g. S.S.F*” and defines:

$$S.S.F = \frac{t_w}{2h_0} \quad (1)$$

$t_w$  is track width and  $h_0$  is the initial height of center of gravity. This model supposes the vehicle as a rigid body. This simple model overestimates the lateral acceleration of vehicle at the rollover threshold [2]. To introduce the effect of roll angle, consider Fig.1.  $\Phi$  is roll angle and could be obtained by equation (2).

$$\Phi = \frac{M_s . h_{roll} . a_y}{k_{\Phi}} \quad (2)$$

The numerator expresses the exerted moment about roll center and the denominator denotes the total rotational stiffness of vehicle (including effect of tire compliance).

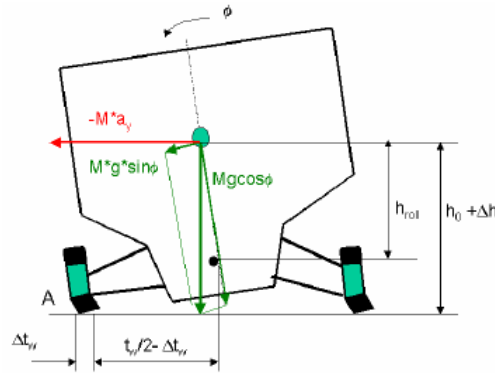


Figure 1: Vehicle Model [2].

## 2.2 Change in track width during steady cornering

Change in track width of the vehicle is caused mainly by two factors, first one lateral compliance of tires and second, kinematics of suspension.

Here it is assumed that the behaviour of tires is linear. So reduction in track width due to tire compliance could be obtained by equation (3).

$$\Delta t'_w = \frac{M . a_y}{k_{lt}} \quad (3)$$

Where  $k_{lt}$  and  $M$  denotes lateral stiffness of tires and total mass respectively. In order to take the second factor into account, one should consider the roll center of tires with respect to body. So tire movement with respect to the body is approximately perpendicular to the line "Ac"; approximately  $\Delta z = \Phi . t_w / 2$ . So  $\Delta t''_w$ , that is increase in track width due to suspension kinematics could be expressed as equation (4).

$$\Delta t''_w = \Delta z . \tan \gamma = \frac{M_s . h_{rollc} . h_{roll} . a_y}{k_{\Phi}} \quad (4)$$

Since  $\Delta t'_w$  and  $\Delta t''_w$  are reduction and increment in track width respectively, total decrease in track width would be:

$$\Delta t_w = M_s a_y \left( \frac{h_{rollc} \cdot h_{roll}}{k_{\Phi}} - \frac{1}{k_{lt}} \right) \quad (5)$$

### 2.3 Effect of jacking forces

Gillespie [4] showed that forces transmitted between wheel and body of the vehicle are equivalent to a force that acts through the line connecting the center of tires contact patch to roll center. The vertical component of this force tends to elevate center of gravity of body and is called “jacking force”. At the threshold of rollover, the lateral inertia force of vehicle must be counterbalanced by the horizontal component of “ $F$ ”, since inside wheels provide no vertical or lateral force.

$$F_v = F_l \cdot \tan \gamma = \frac{2M_s h_{rollc} a_y}{t_w} \quad (6)$$

Where  $F_v$  and  $F_l$  denote components of  $F$  in vertical and lateral directions respectively. Jacking force results in change in height of center of gravity of the body. This change in height is:

$$\Delta h = \frac{F_v}{k_{vt}} = \frac{2M_s h_{rollc} a_y}{k_{vt} t_w} \quad (7)$$

Where  $k_{vt}$  denotes total vertical stiffness of vehicle suspension. Substituting this amount of acceleration in equation (6) yields:

$$\Delta h = 0.8(h_{rollc} / h_0)(M_s g / k_{vt}) \quad (8)$$

### 2.4 Lateral acceleration at the rollover threshold

Lateral acceleration at the rollover threshold is obtained by taking moment about outside tires. Referring to equation of equilibrium, the following relation can be written:

$$\sum T_A = M a_y (h_0 + \Delta h) - M g \cos \Phi (t_w / 2 - \Delta t_w) + M g \sin \Phi \cos \Phi h_0 = 0 \quad (9)$$

Letting  $\sin \Phi = \Phi$  and  $\cos \Phi = 1$  for small roll angle, equation (9) can be written down as:

$$a_y = \frac{g \cdot SSF}{[1 + \Delta h / h_0 + M_s g h_{roll} (1 - h_{rollc} / h_0) / k_{\Phi} + M g / (k_{lt} h_0)]} \quad (10)$$

In the following, the relation for lateral acceleration at the rollover threshold in steady state cornering is going to be developed into the transient state to take account of effect of soil.

## 3. THE ROLLOVER THRESHOLD AT TRANSIENT MODE

In order to develop equation (10) from steady state to transient state in which lateral inertia force is not constant, a simplified model of vehicle is used (Fig.2). Equation of motion for this

model is stated in equation (11). Thereby the effect of varying tangential force exerted on tires by soil is introduced to the equation (10).

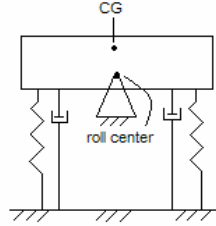


Figure 2: Vehicle model

$$I_s \frac{d^2 \Phi}{dt^2} + C_\Phi \frac{d\Phi}{dt} + k_\Phi \Phi = T(t) \quad (11)$$

Where  $I_s$  denotes the inertia of moment of the sprung mass about center of gravity, and  $C_\Phi$  and  $k_\Phi$  denotes total roll damping and total roll stiffness of the vehicle and  $T(t)$  is the moment exerted about the roll center of the body.  $T(t)$  is generally a function of time which for a especial case of steady-state cornering it is constant. For example for a vehicle which passes from an ideally smooth road to a rough one (during cornering),  $T(t)$  exhibit a step function as depicted in Fig.3.

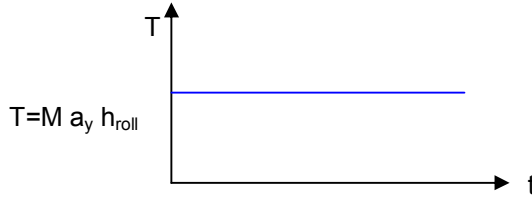


Figure 3: Exerted moment on model during transition from smooth to rough road

Assuming applied moment as a step function, solving of equation (11) for  $\Phi$  is a curve oscillating around  $\Phi_s$  (steady-state roll angle which is obtained by substitution of equation (10) into equation (2)). Therefore,  $\Phi_{max}$  would be:

$$\Phi_{max} = \Phi_s (1 + \Delta_{os}) \quad (12)$$

Where,  $\Delta_{os}$  could be obtained by equations (13) and (14):

$$\Delta_{os} = e^{\left(\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}\right)} \quad (13)$$

$$\zeta = \frac{C_\Phi}{2\sqrt{I_s k_\Phi}} \quad (14)$$

Development of the steady-state relation (equation (10)) to a relation that also consider the effects of step moment function (not constant moment) is done by multiplying  $h_{roll}$  in equation (10) by  $(1+\Delta_{os})$ . It could be conformed by deriving equation (10) considering  $\Phi_{max}$  rather than

$\Phi$ .  $T(t)$  as a step function of time introduces an example of developing equation (10) to non-steady state occasions.

### 3.2 Effect of soil on lateral acceleration at the rollover threshold

The process of considering the soil impact is identical to the process introduced in the step function  $T(t)$  in previous section. Once again equation (11) must be solved by substituting appropriate  $T(t)$ .

## 4. DIRECT STUDY OF ROLLOVER

During maneuvering or turning the vehicle around a curve, there exist the effect of tire sinkage through the soil and consequently the compaction of soil behind the outside tire. Therefore, in this article, we consider this situation similar to a tripped rollover and we just consider the effect of soil compaction, which leads to rollover of an off-road vehicle (Fig. 4).

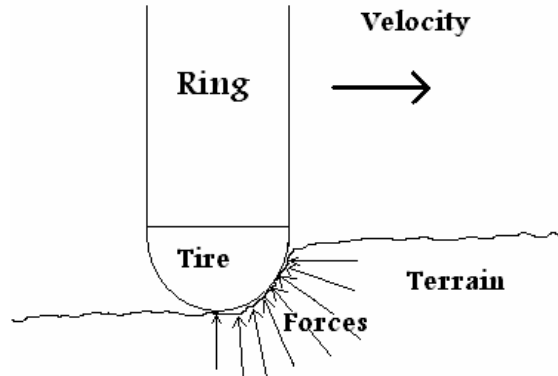


Figure 4: Graphic simulation of tire while turning on a given terrain.

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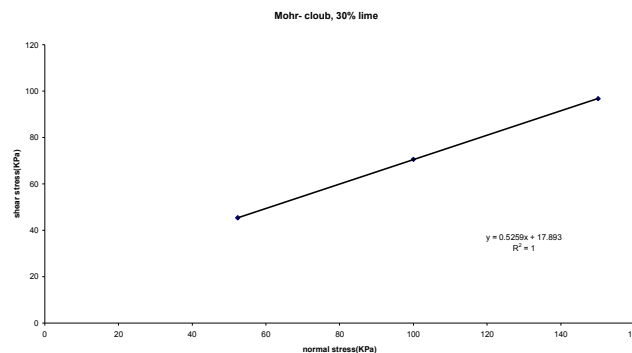


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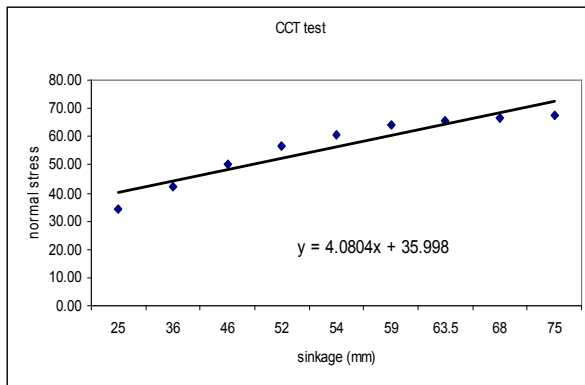


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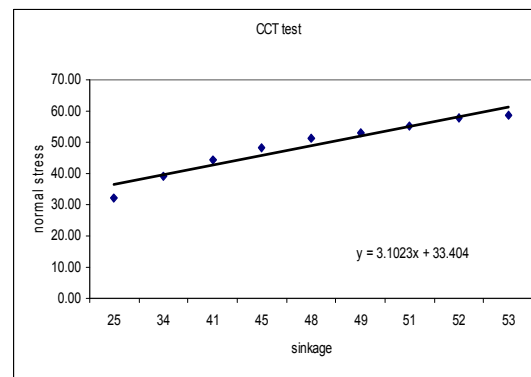


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