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IMPACT MODELLING OF SPOT-WELDED COLUMNS FABRICATED WITH ADVANCED HIGH STRENGTH STEELS

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ABSTRACT - In order to increase vehicle fuel efficiency while improving safety and performance and maintaining affordability, the global steel industry has initiated the use of advanced high strength steels that ultimately result in the design of stronger, lighter and more energy efficient vehicles. Tubular steel columns are extensively used in the automotive body structure due to their inherent capacity to absorb energy on impact. Hence, the objective of this research work is to study the crashworthiness performance of spot-welded columns made from advanced high strength steels. It is shown that numerical results of the developed robust finite element model give fairly good agreement with experimental data in terms of collapse profile, deformed column shape, final crush length, and impact peak force. Throughout the investigation, the finite element model permits the study of several structural and material variables that can be validated by a moderate set of destructive tests. Moreover, the current finite element crash model and the findings in this work can eventually be used to improve the crashworthiness efficiency of steel column specimens and to help meet society's demands for affordable, fuel efficient, environmentally responsible, and safe vehicles.

TECHNICAL PAPER

Introduction

Current extensive research in automobile crash performance is intended to protect vehicle occupants, pedestrians and property by helping to reduce the occurrence of an accident and diminish the effects of such possible car crashes. In order to mitigate the damage caused to the passengers during vehicle impacts, the design of better energy absorbers has received much attention over the decades [1]. Hat-type cross-section columns have been implemented in several automotive systems for absorbing energy during impact events. These types of elements can be found in front and rear car rails, crossmembers in the B-pillar structure, bumpers and B/C pillar reinforcements [2-5]. The use of advanced high strength steels (AHSS) for these columns is an effective solution for satisfying the demands of safety and fuel consumption due to their inherent crashworthy properties and lightweight [6].

Ben-Yahia et al., [2] evaluated the crashworthiness performance of advanced high strength steels (AHSS) on single-hat and double-hat columns made of DP and HSLA steels. A finite element model was developed and the numerical computations for the dynamic crushing of hat-type columns compared well with the experimental data available for different impact conditions. Maximum dynamic crush load, final crush length of the columns and force-time history were reported. The study in this work is the continuation of the above investigation, where the tied constraints between the parts of the column have been replaced by spot welds to predict the buckling modes in the static analysis. New enhanced contact interactions between the hat and plate structures have been implemented as well as incorporation of spot-weld failure.

The main objective of the present work is to study the crashworthiness characteristics of columns made from advanced high strength steels. To carry out this investigation, the current research aims to develop and validate a robust finite element crash model that can be used to evaluate the collapse response of spot-welded hat-type cross-sectional columns. Experimental tests, performed at the laboratory facilities of Stelco Inc. Hamilton, Ontario are used to validate the finite element simulations. In the present study, special interest is given to investigate the effect of initial geometry imperfections on the collapse behavior and deformed shape of the columns. The FEA model developed in this work, allows for the assessment of the effects of various geometrical and material parameters on crushing of AHSS columns at different impact velocities. It also permits us to evaluate the influence of spot-weld failure on structural crashworthiness in both single hat and double hat configurations. Consequently, with the ability of numerical computer simulation to predict the collapse and postbuckling response of single-hat and double-hat, spot-welded steel specimens, this type of analysis is expected to be very useful in the design and analysis of energy absorber elements before actual destructive tests are to be used.

Experiments

Materials: In the present work, high strength steel materials HSLA and dual phase (DP) were selected for drop weight experiments in order to investigate the crashworthiness performance in single-hat and double-hat longeron column specimens. The mechanical properties of HSLA and the two variants of DP600 high strength steels are listed in Table 1 [2].

Table 1: Chemistry and static mechanical properties of steel materials considered in this study.

STEELS			*CHEMISTRY (wt%)*				*STATIC MECHANICAL PROPERTIES*				
Label	Name	Type	C	Mn	Si	Other	Young Modulus E (GPa)	Yield Strength YS (MPa)	Tensile Strength TS (MPa)	Total Elongation El (%)	n-value
DP 600/400	SteelPhaseII600	DP	0.06	1.90	0.08	Mo	207	400	620	23	0.13
DP 600/300	SteelPhaseII600	DP	0.06	1.90	0.08	Mo	207	300	620	25	0.15
HSLA	Stelmax50	HSLA	0.07	0.90	0.13	Nb	207	360	430	35	0.20

Specimens: The geometries of the single-hat and double-hat columns considered in this work are shown in Figure 1. The corresponding dimensions are reported in Table 2.

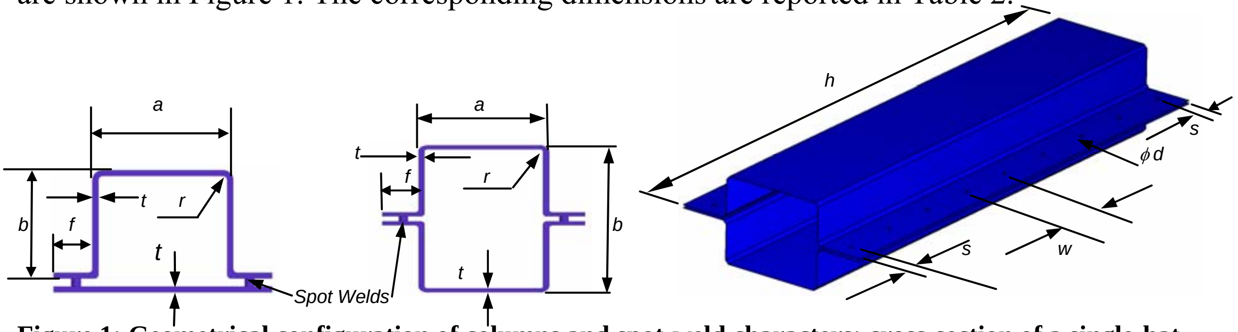


Figure 1: Geometrical configuration of columns and spot-weld characters: cross-section of a single-hat specimen, cross-section of a double-hat specimen, and spot weld arrangement demonstrated in a double-hat specimen.

Table 2: Geometry of hat-type sections.

Cross-type section	Wall thickness t (mm)	Width a (mm)	Width b (mm)	Height h (mm)	Flange f (mm)	Inner-outer rolling radius r (mm)	Spot-weld spacing w (mm)	Edge spacing s (mm)	Spot-weld diameter d (mm)
Single-hat	1.7	63.6	52.6	300	22.2	2	39	13.5	5.4
Double-hat	1.7	63.6	50.8	300	30.1	2	39	13.5	5.4

Experimental method: The dynamic crash tests on hat-type cross-section columns were undertaken on a drop weight machine with a maximum drop height of 4.27 m. The columns were crushed with impactor weights of 181.4 kg and 148.2 kg and drop heights of 2.95 m and 4.27 m, respectively. A 222.4 kN (50 klb) load cell was used to measure force-time histories of experiments carried out with the 181.44 kg impact mass [2]. A load cell of 444.8 (100 klb) was instrumented to measure the force and axial crush response of columns subjected to an impact mass of 148.2 kg . The test specimens were positioned at the lower end by a sleeve cap in order to prevent the loaded ends from moving laterally, while offering little resistance to out-of-plane rotation. The impacting mass hits the cross-head of the specimen in a close approximation to a freestanding condition. The experimental tests reported herein were conducted by and in the laboratory facilities of Stelco Inc. (Hamilton, Ontario, Canada). After performing the drop tower destructive experiments, the final (permanent) reduction in axial length of the specimens was measured manually.

Numerical Model

The nonlinear finite element commercial package, ABAQUS, is employed to simulate the dynamic crush behavior of hat-type columns fabricated from advanced high strength steels. The crashworthiness study involves two analysis runs with basically the same model definition. ABAQUS/Standard is used to establish the probable collapse modes of the column and ABAQUS/Explicit is used to perform the post buckling analysis. The first ten buckling modes and eigenloads of the columns are computed by running a static linear buckling analysis in ABAQUS/Standard. The nodal coordinates of the ten modes predicted by the eigenvalue analysis are stored. Under quasi-static load conditions, it is expected the post buckling deformation to resemble the eigenmode shape corresponding to the lowest eigenvalue, unless the lowest eigenvalues (eigenloads) are closely spaced, in which case the post buckling deformation is probable to be a mixture of the lowest eigenmodes. ABAQUS/Standard normalizes the eigenmodes, thus the maximum deformation of the structure in the length of units of the analysis (millimeters in this case) is equal to 1. Imperfections are introduced by scaling and adding the ten lowest symmetric buckling modes to the perfect geometry to create the perturbed mesh. Since presumably an imperfection in the shape of the lowest mode would be the most critical, the largest scale factor is given to the first mode, and the scaling factor monotonically decreases as the mode number increases. ABAQUS/Explicit reads the buckling modes, scales them, and uses them to perturb the nodal coordinates of the column. Perturbation magnitudes are chosen in such manner that the mesh is modified with a deformation pattern that will ultimately allow the explicit crush deformation to proceed correctly.

Spot Welds Modelling: Two FE numerical models were developed for simulating the collapse response of crashworthy columns subjected to axial impacts using ABAQUS mesh independent spot welds-Fastener and element connection type-Weld. The mesh-independent fastener method consists of specifying the location and the properties of the spot welds in a simple manner regardless of the mesh configuration. The mesh-independent spot welds are modeled as rigid constraints, therefore this method does not allow for spot-weld failure. On the other hand, the element connection type-Weld capability allows for welds to break, though this procedure is dependant on the mesh configuration. The spot-weld fails entirely once the constraint forces exceed the failure criterion [7].

Dynamic Analysis: The impact analysis is carried out using the explicit dynamics FE code ABAQUS/Explicit. The finite element model comprises three structures: a fixed rigid plate, the hat-type column, and a movable rigid plate (impactor). A point mass is attached to the rigid movable plate, and an initial velocity is assigned to it. The momentum due to this initial velocity will cause the impactor plate to strike the steel column, which will ultimately crush against the fixed rigid plate. Figure 2 illustrates the assembly for the dynamic impact analysis demonstrated in a single hat specimen. The finite element model of the hat-type cross-section column is discretized with three-dimensional shell element S4RSW. The S4RSW is a 4-node reduced integration, doubly curved shell with hourglass control element and has six degrees of freedom at each node (three translations and three rotations).

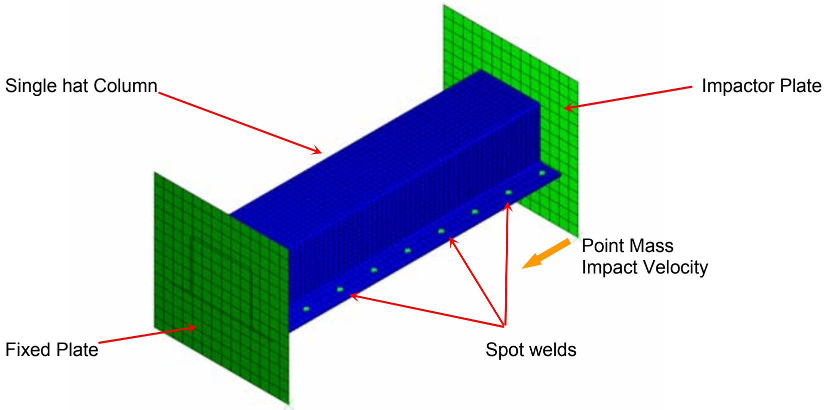


Figure 2: Explicit finite element model demonstrated in a single-hat specimen.

Material Constitutive Model: The steels are considered to be elastic-plastic materials and the Johnson-Cook constitutive model, given by Eq. [1], is used to characterize the plastic behaviour of the materials [8]. The model includes a strain rate sensitivity component, which plays an important role in vehicle impacts. The values of the material parameters for the steels considered in this work where taken from Ben-Yahia F., et al. [2] who determined the model constants by performing quasi-static tension tests on specimens of the materials.

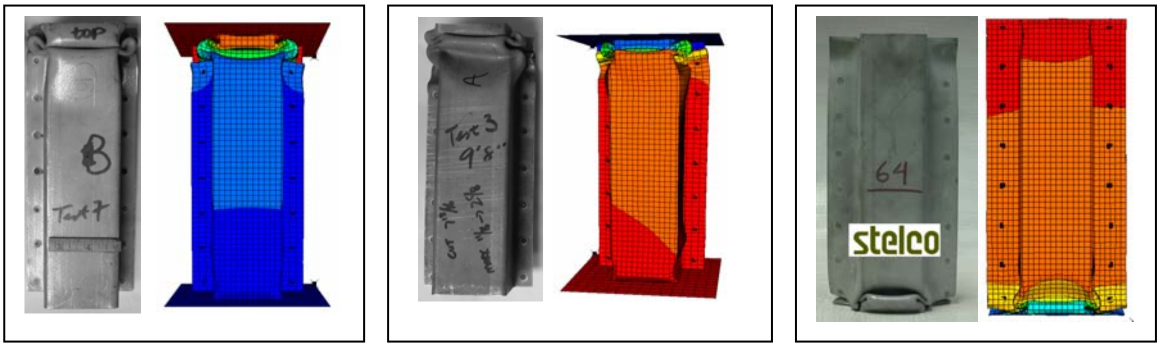
$$\sigma = [A + B\varepsilon_p^{n'}][1 + C \ln \frac{\dot{\varepsilon}_p}{\dot{\varepsilon}_o}][1 - (\frac{T - T_0}{T_{melt} - T_0})^{m'}] \qquad (1)$$

were σ is the equivalent stress, ε_p is the equivalent plastic strain, ε'_o is the reference strain rate (typically taken to be 1 s^{-1}), ε'_p is equivalent strain rate and T is the temperature of the specimen. T_{melt} and T_o are the melt and reference temperature, respectively. The constitutive parameters A, B, C, n' and m' in the Johnson-Cook equation are listed in Table 3.

Table 3: Johnson-Cook model material constitutive parameters for steels examined.

STEEL	***A* (MPa)**	***B* (MPa)**	***C***	***n´***	***m´***
DP 600/400	300	630	0.030	0.30	1.0
DP 600/300	270	875	0.080	0.36	1.0
HSLA	344	200	0.025	0.33	1.0

Modeling the Imperfect Column: A linear perturbation procedure was performed in ABAQUS/Standard to obtain the first eigenmodes of the hat-type columns, geometric imperfections were introduced considering only the first symmetric mode shapes, and finally a set of scale factors was determined based on direct comparison of the numerical results with the experimental data for one particular impact condition. As commonly adopted by several authors [7], the scale factor is usually taken to be some fraction of the shell thickness of the structure. In this work, different sets of scale factors are chosen with the lowest or first symmetric lowest eigenmodes scaled to 1%, 2%, 10% and 100% of the shell thickness. The scaling factor monotonically decreases as the mode number increases, the specific values are chosen arbitrarily. A scale factor set formed mainly with 1% of the first symmetric eigenmode shape and 0.25% of the second gives better agreement with the experimental test results in terms of both height reduction and peak force. This scale factor set is selected to incorporate geometric imperfections into all FE computer models (small variation of the scale factor values of the higher eigenmodes is found to have an insignificant effect on the postbuckling behavior). In order to verify that the degree of geometric imperfections introduced into the numerical model makes the deformation proceed correctly, a comparison of the crushed shape of the crashworthy structure and the computed contour plots of the axial displacement is illustrated in Figure 3. Details of the steel used, column geometry and impact condition are given in the figure. Based on observation of Figure 3, the computer deformed shapes of the columns agree well with the experimental collapse profiles.



a) **Geometry:** Single hat column
Steel: DP600/300
Impacting mass: 181.44 kg
Impact velocity: 7.6 m/s

b) **Geometry:** Single hat column
Steel: HSLA
Impacting mass: 181.44 kg
Impact velocity: 7.6 m/s

c) **Geometry:** Double hat column
Steel: DP600/400
Impacting mass: 148.2 kg
Impact velocity: 9.15 m/s

Figure 3: Verification of deformed shapes (deformed specimens courtesy of Stelco Inc.)

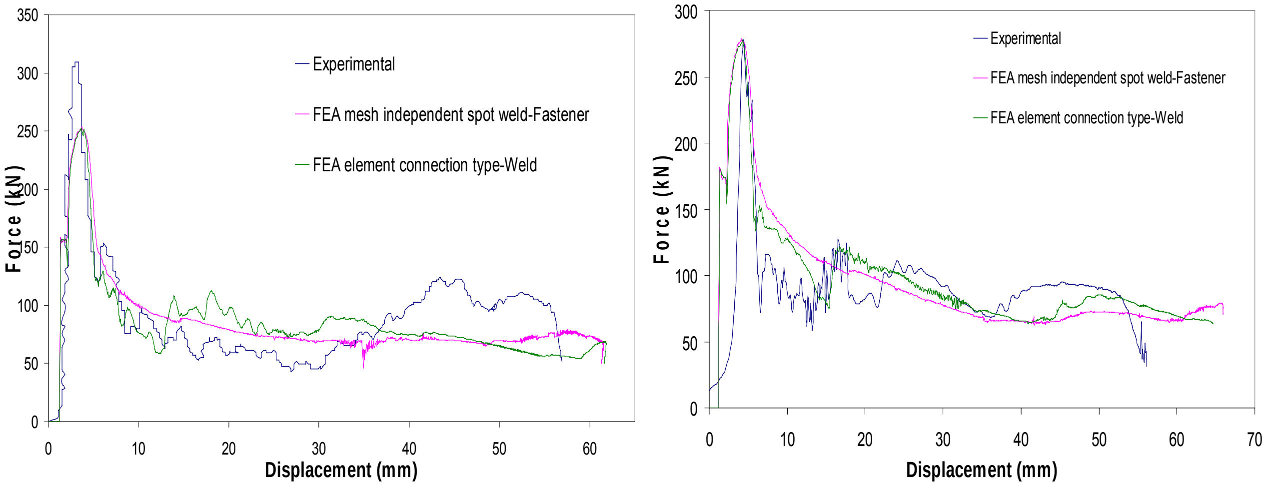
Numerical Results

Numerical Results for Single Hat Columns**:** two quantities of particular interest in the dynamic crushing analysis of hat-type columns are the permanent axial reduction of the column's length and the interface force between the impactor and the top of the specimen as a function of the axial displacement. The computed peak load, mean load, and height reduction of DP600/400, DP600/300 and HSLA steel columns subjected to a mass of 148.2 kg impacting at a velocity of 9.15 m/s are listed in Table 4. As seen in the table, the computed results compare well with the experimental data for both ways of modeling the spot welds of the column. The predicted peak load is lower than the average peak load measured in experiments for the DP600/400 steels, however the value is above the lowest experimental peak load recorded (250 kN) and therefore the numerical results are acceptable. A considerable difference between the peak load is observed for the HSLA column. Such a discrepancy may be directly attributed to the upper yield stress at a small deformation in the HSLA true stress - true strain curve which can not be captured in the Johnson-Cook constitutive model. However, the mean crushing force from the test results and the numerical predictions show good agreement between them.

Table 4: Single hat column struck by 148.2 kg at 9.15 m/s. (experimental data courtesy of Stelco Inc.).

Material		Peak Load (kN)	Height Reduction (mm)	Mean Load (kN)
DP600/400	Experimental	281.25 ± 31.25	67.81 ± 3.13	96.33
	FEA-mesh independent spot weld-Fastener	257.99	67.09	92.15
	FEA-connector element type-WELD	260.64	69.99	89.03
DP600/300	Experimental	275 ± 31.25	60.03 ± 2.81	81.88
	FEA-mesh independent spot weld-Fastener	278.87	63.46	83.72
	FEA-connector element type-WELD	277.89	64.89	83.05
HSLA	Experimental	353.12± 39.06	81.25 ± 3.75	70.67
	FEA-mesh independent spot weld-Fastener	217.99	83.35	67.71
	FEA-connector element type-WELD	214.83	92.14	65.61

Experimental and numerical load-displacement characteristics correlate very well as depicted in Figure 4. From Figure 4-a it can be seen that an exact match between the predicted and experimental curves is not achieved during the initial part of the crushing (where the peak load is presented). However, after the peak load the curves show good correlation. It is observed from Figure 4-b, that the FE results closely agree with the experimental peak load for the particular test shown.



a) **b)**

Figure 4: a) Force-displacement history for single hat column, m=181.44 kg, v=7.6 m/s, material DP600/400. b) Force-displacement history for single hat column, m=148.2 kg, v=9.15 m/s, material DP600/300 (experimental data courtesy of Stelco Inc.).

Test results of the axial crushing of specimen fabricated from DP600/400, DP600/300 and HSLA steel struck by a drop-weight of 181.44 kg moving at an impact velocity of 7.6 m/s, and the numerical predictions are listed in Table 5. It can be concluded that the mean load predictions obtained from the finite element analysis are found to be in good agreement with the experimental results.

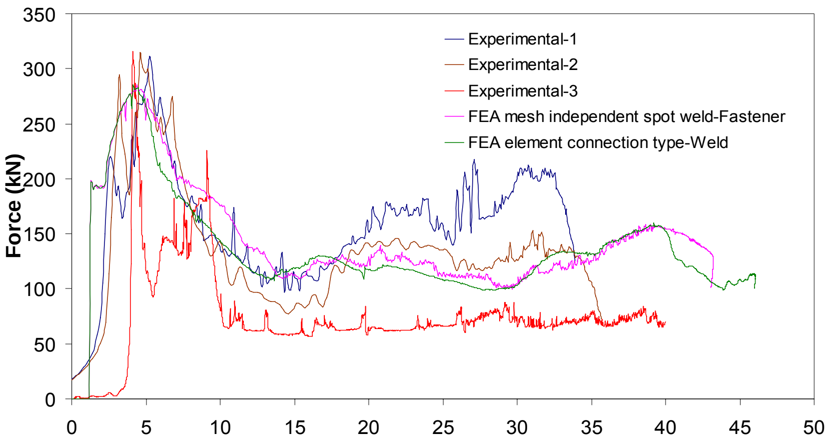
Table 5: Single hat column struck by 181.44 kg at 7.6 m/s (experimental data courtesy of Stelco Inc.).

Material		**Peak Load (kN)**	**Height Reduction (mm)**	**Mean Load (kN)**
DP600/300	Experimental	309.24	N/A	83.93
	FEA-mesh independent spot weld-Fastener	257.83	60.74	85.84
	FEA-connector element type-WELD	252.45	60.49	76.37
DP600/400	Experimental 1	326.85	N/A	85.83
	Experimental 2	307.78	N/A	79.23
	FEA-mesh independent spot weld-Fastener	269.44	55.99	80.62
	FEA-connector element type-WELD	263.30	54.06	84.56
HSLA	Experimental 1	339.45	N/A	62.03
	Experimental 2	306.74	N/A	55.36
	FEA-mesh independent spot weld-Fastener	215.43	69.72	62.64
	FEA-connector element type-WELD	213.37	78.84	63.03

Numerical results for double hat columns: Experimental and numerical results for the double hat column made from DP600/400 steel are shown in Figure 5-a. Three specimens were impacted with a striking mass of 148.2 kg traveling at a velocity of 9.15 m/s. The force versus axial displacement curves are depicted in Figure 5-b. From Figure 5-a it can be seen the FEA computer simulations underpredict the peak load (30 kN approximately, which represents ≈ 10% underprediction of the experimental peak load). However they estimate satisfactorily the experimental mean load. Computer force-displacement histories compare better with the recorded experimental-2 curve, as seen in the figure.

	Peak Load (kN)	Height Reduction (mm)	Mean Load (kN)
Experimental	313.58	37.55[1]	125.51
FEA-mesh independent spot weld-Fastener	285.20	41.64	139.83
FEA-connector element type-WELD	284.67	45.25	127.21

a)



b)

Figure 5: a) Experimental and numerical results for double hat column struck by 148.2 kg at 9.15 m/s , material:DP600/400, b) Force-displacement history (experimental data courtesy of Stelco Inc.).

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[1] This value was obtained as the average of three manual measurements of the column crush length on top view pictures of deformed double hat specimens.

Test data for double hat structures fabricated from DP600/300 and HSLA steels and struck by an impactor of 148.2 kg moving at a velocity of 9.15 m/s are not available. However, the computer simulation results are shown in Figure 6. Experimental work was not conducted for double hat columns impacted by a 181.44 kg mass traveling at a velocity of 7.6 m/s. However, FE numerical results for both ways of modeling the spots welds of the structure are shown in Figure 6. Computer crush length results for double hat columns are summarized in Figure 6. It has been found that the crush length is shortest for the DP600/300 steel, followed by DP600/400 and the HSLA, indicating again the greater energy absorption of the DP steels. This correlates with the results for the single hat geometry. It is also noted that the crush length of the DP and HSLA steels becomes shorter as the crash energy is reduced and it is more pronounced in the HSLA than the DP steels, similar to the single hat geometry. Peak force and mean force are lower in the HSLA than the DP steels. Similar post-buckling behaviour was found for the single hat columns. It is also noted that the peak force and mean force are quite similar for both cases of input energies.

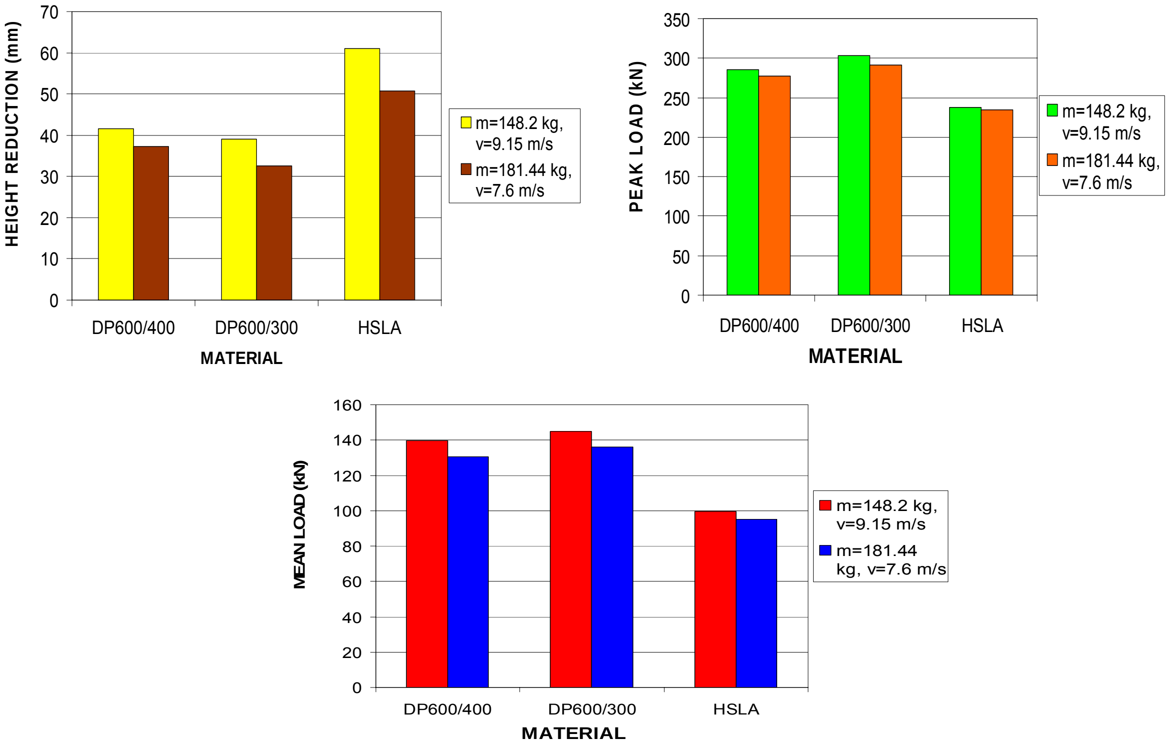
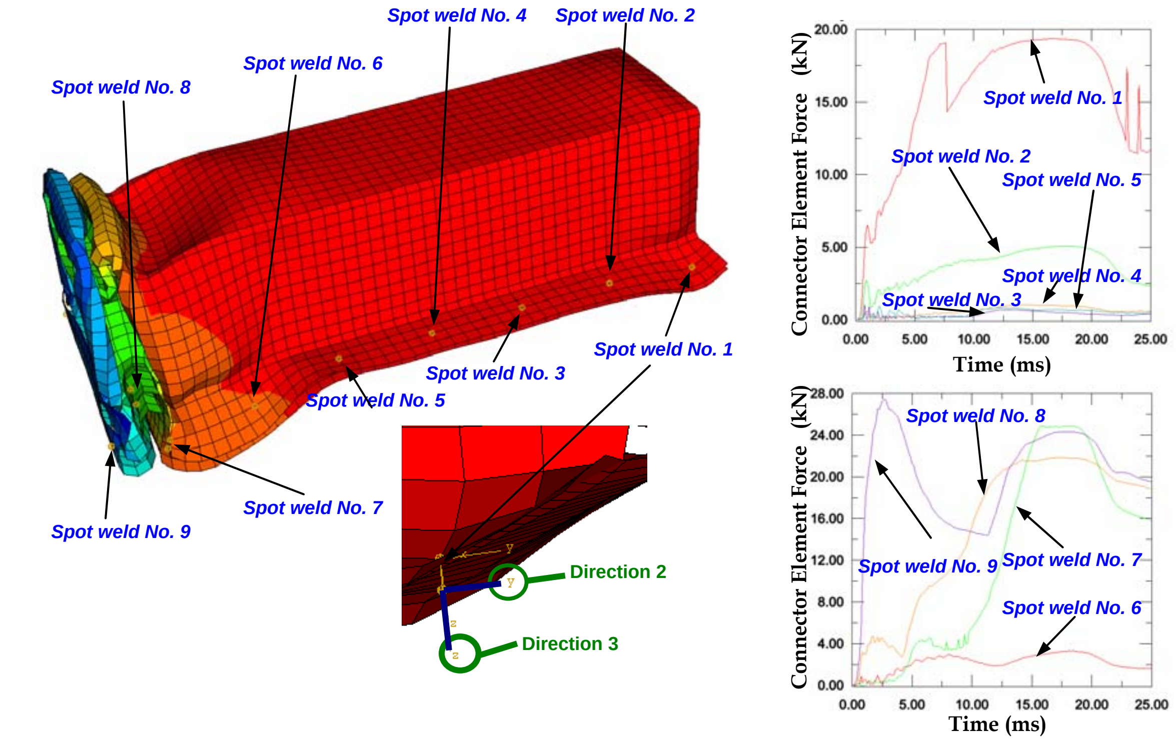


Figure 6: Height reduction, Peak load and Mean load characteristic for double hat columns. Numerical results of the FEA model using mesh independent spot welds-Fastener.

A comparison between the results shown in Tables 4 & 5 and Figure 6 indicates the crashworthy efficiency of the double hat columns. The axial displacement of this geometry is significantly less than the single hat and consequently better in terms of energy absorption. However, it must be noted that the cross sectional area of the double hat column is approximately 8% larger than the single hat structure. On the other hand, the peak load is slightly lower for the single hat columns, which is desirable.

Spot Weld Failure Computer Results: Vehicle designers keep in mind two parameters, the crush length of the energy absorbing devices and the peak force transferred to other structures. The axial crushing displacement of the column is crucial to maintain space integrity in the occupant compartment. The durability and structural integrity of hat-type welded structures are generally controlled by the strength of the spot welds which commonly fail under combined loads during a car impact. Therefore, a failure criterion is necessary for the structural integrity design of the vehicle. Although the FE model predictions using mesh independent spot weld-Fastener agree closer than the FE model using connector element type-Weld, the first method models rigid spot welds and does not allow for spot weld failure. The Connector Element Force capability in ABAQUS is used to evaluate the forces acting on the spot welds. It is found that the most significant forces are acting on each spot weld in the direction normal to the surface of the flange of the column (Direction 3, local coordinate systems, see Figure 7-a). Figure 7-b illustrates the carry on capacity on each spot weld for the single hat column made of DP600/400 steel and subjected to a striking mass of 148.2 kg traveling at a velocity of 9.15 m/s. Because of the symmetry of the structure, only the 9 spot welds located on one side of the flange are shown.



a)

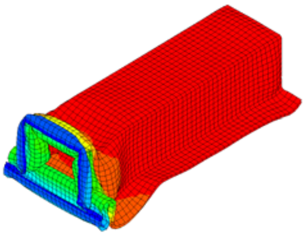
b)

Figure 7: a) Deformed shape for single hat column, m=181.44 kg, v=7.6 m/s, material DP600/400. b) Connector element force acting on the spot welds of the column, directin-3 .

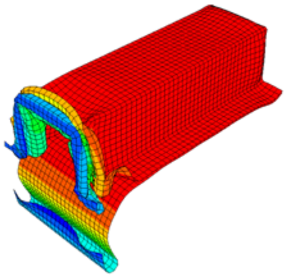
It can be observed from the above figure that spot weld numbers 1, 7, 8 and 9 experience higher loads during the crushing process in comparison to spot welds 2, 3, 4, 5 and 6. To see the effect of spot weld failure, different tensile limit loads are implemented. Figure 8-a illustrates the failure time for each spot weld, when varying the tensile maximum load from 15 kN to 5 kN. It should be observed that only 3 spot welds experience failure with a 15 kN tensile limit load (spot welds numbers 7, 8 and 9) and the number increases as the tensile load decreases, indicating the decrease of the structural integrity of the column. It is also noted that the time to failure for a particular spot weld decreases as the tensile limit load decreases, especially for those nodes located close to the folding end of the column (spot welds 7, 8 and 9). The contour plots for the axial displacement of the column modeled with rigid spot welds and breakable spot welds with 5 kN tensile failure load are shown in Figure 8-b and 8-c respectively. As can be seen, the plate element deforms and remains attached to the hat component during the dynamic collapse of the single hat column modeled with rigid spot welds. On the contrary, the plate deforms and tends to separate from the hat component during the dynamic crushing of the column modeled with breakable spot welds with a maximum capacity of 5 kN tensile load.

Spot-Weld Number	TIME TO FAILURE (ms)				
	Tensile Load 15 kN	Tensile Load 12.5 kN	Tensile Load 10 kN	Tensile Load 7.5 kN	Tensile Load 5 kN
1	-	11.6	10.9	-	0.8
2	-	-	-	-	-
3	-	-	-	-	-
4	-	-	-	-	-
5	-	-	-	-	-
6	-	13.5	12.9	10.9	19.4
7	13.6	13.5	11.8	8.0	3.9
8	4.0	6.8	3.4	4.3	1.4
9	1.0	0.9	0.8	0.7	0.6

a)



b) Rigid spot welds



c) Breakable spot welds

Figure 8: a) Time to Failure prediction, single hat specimen made of DP 600/400, m=148.2 kg traveling at a velocity of 9.15 m/s. b) & c) Deformed shapes for a single hat column modeled with rigid spot welds and breakable welds using the connection element type-Weld capability in ABAQUS.

Figure 9 displays the end-shortening and mean load characteristics for a DP600/400 single hat column struck by an impact mass of 148 kg traveling at a velocity of 9.15 m/s. A difference of more than 10 mm crush length is found between the unbreakable spot welds computer model and the one with spot welds constrained to withstand a maximum tensile load of 5 kN. The predicted peak load in all cases is 258 kN. This is reasonable since the peak load is usually manifested within the first 4 milliseconds when the spot welds have not yet failed. It can be seen that the mean load decreases as the tensile limit load decreases. A failure spot weld analysis like the one performed in this work could be extremely relevant from the vehicle design stand point. The FE model can be effectively used for predicting dynamically loaded spot welded failure for different impact conditions.

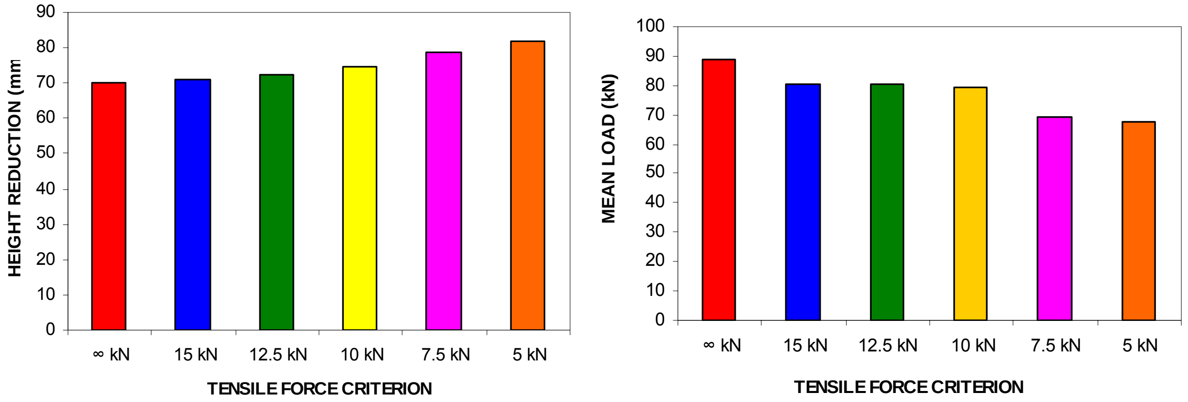


Figure 9: Height reduction and mean load for a single hat column modeled with rigid spot welds and breakable spot welds using the connection element type-Weld capability in ABAQUS. The column is fabricated from DP600/400, striking mass of 148.2 kg at 9.15 m/s.

Conclusions

It is shown that the finite element computer models predict deformed shapes that closely agree with those seen in the post-test observations. It is found that the numerical model correctly predicts the mean crushing force, impact peak load and crushed column length. In addition, by examining the force versus axial displacement curves, it can be concluded that the computer simulation results provide good predictions of the history results obtained in destructive drop tests. Significant differences in crash energy capacity among the high strength steels can be evaluated through the end-shortening of the member. It is observed that the dual phase steels (DP) exhibit better energy absorbed capacity than the high strength low allow steel (HSLA) for both geometric configurations. Incorporation of spot weld failure into the model allows the prediction of the dynamic collapse response of hat-type cross-section spot-welded structures when considering an ultimate load capacity on the spot welds. It is expected that both robust numerical models will potentially be used to continue to study the effects of structural geometry and material parameters while varying the impact weights and velocities, minimizing the extent of physical tests and the costs associated.

Acknowledgments

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