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Simulations for the use of GPS compensated sensors for vehicle dynamic systems control.

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ABSTRACT

Assuming a five degree of freedom non-linear bicycle model, the transient response of the vehicle is simulated assuming the presence of stochastic stationary and time varying noise and disturbance properties. Various combinations of linear and non-linear tyre model in the plant and the Extended Kalman Filter (EKF) design are simulated in order to investigate the effects of plant uncertainty on EKF estimation performance. It is found that the EKF is only able to generate state estimates that track the true response when the same tyre model is used in both the plant and EKF design. Therefore, varying approaches from the literature for robust EKF design using parameter identification are discussed with a view to future work.

INTRODUCTION

The objective for this paper is to investigate the possibilities for vehicle dynamic applications given that full state measurement is available [1]. At present, some state estimates are obtained by integrating inertia sensor information. However, this data is subject to uncertainties and errors [1, 2, 3], which will be amplified after the process of integration. As will be discussed, these issues can be addressed by compensating the inertia sensor data with GPS (Global positioning system) information [4]. With such an integrated sensor approach, systems will be potentially more robust and able to deal with a continuously changing operational environment.

In many cases, state variables cannot be measured directly, and even where direct measurement is possible, these signals will be subject to stochastic noise. In both situations, the state observer technique is applied to estimate and filter the state variables respectively before they are used by a controller. It could be assumed that the estimator is operating for a linear plant where the parameters of the dynamic system are assumed to be constant. In real situations however, the plant is non-linear and the parameters are changing. Examples of changing parameters would include the tyre characteristics, the load on the car changing and so on. Therefore the state observer must be adaptive and hence identify these parameters in real time in order to cope with the non-linear dynamic plant [5]. As will be discussed later in the paper, the plant model is linearised about a given operating point when designing the KF (Kalman Filter). In order to cope with a non-linear plant however, multiple plant linearisations will be required as the plant operating point changes.

As discussed previously, the estimator used in this simulation work is an EKF. In this work the EKF is implemented assuming the full state variable vector is either measured directly using a GPS & INS sensor system [6], or indirectly with some of the state variables observed from lateral acceleration, yaw rate and rotational wheel spin velocity measurements [7].

Due to inherent inertial sensor errors, these observed state variables are, in practice, themselves subject to additional filtering by a separate KF. In summary therefore, GPS can offer a solution that provides an external global reference to correct for the inherent errors in INS sensor measurements. Such a capability has been commented on in the literature where it is stated that the low frequency errors from the GPS measurements are compensated for by the INS measurements, and the high frequency errors inherent in the INS sensor system are compensated for using the GPS measurements [8].

THE VEHICLE MODEL

The vehicle model used in this paper for the EKF design and plant model is a five degree of freedom lumped parameter ‘bicycle model’, see Figure 1. The coordinates in the body centred inertial reference frames are based on the SAE sign convention [9]. The active degrees of freedom in the model are the longitudinal, lateral and yaw motions together with the lumped front and rear wheel spin rotations, x, y, ψ, θ_f and θ_r respectively.

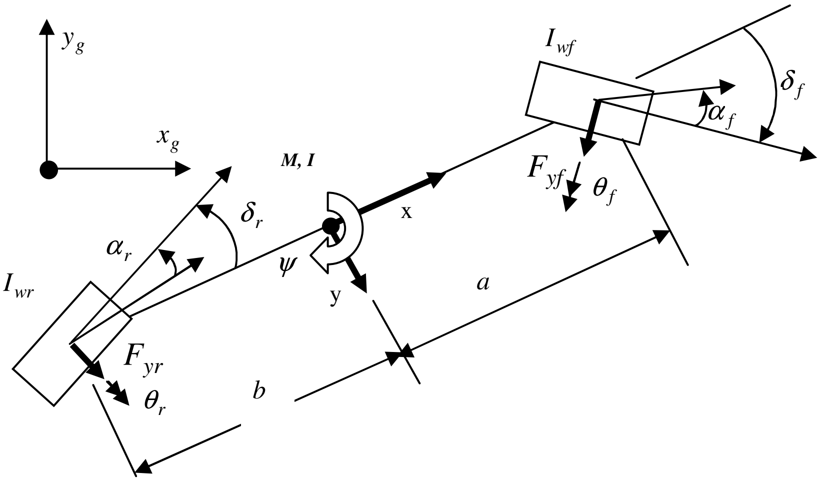

Figure 1: Schematic of five degree of freedom vehicle model.

Using the Newtonian approach the five non-linear equations of motion in the body centred inertial frame are derived.

$$\ddot{x} = \frac{1}{M}\left[\begin{array}{l} M\dot{y}\dot{\psi} + C_{xf}\left(\frac{\dot{\theta}_f R - \dot{x}}{\dot{x}}\right)\cos\delta_f + C_{yf}\left(\tan^{-1}\left(\frac{\dot{y}+a\dot{\psi}}{\dot{x}}\right) - \delta_f\right)\sin\delta_f \\ \quad + C_{xr}\left(\frac{\dot{\theta}_r R - \dot{x}}{\dot{x}}\right)\cos\delta_r + C_{yr}\left(\tan^{-1}\left(\frac{\dot{y}-b\dot{\psi}}{\dot{x}}\right) - \delta_r\right)\sin\delta_r - \frac{C_d^x A_x}{2\rho}\dot{x}^2 \operatorname{sign}(\dot{x}) \end{array}\right] \tag{1}$$

$$\ddot{y} = \frac{1}{M}\left[\begin{array}{l} C_{xf}\left(\frac{\cos\delta f\dot{\theta}_f R - \dot{x}}{\dot{x}}\right)\sin\delta_f - C_{yf}\left(\tan^{-1}\left(\frac{\dot{y}+a\dot{\psi}}{\dot{x}}\right) - \delta_f\right)\cos\delta_f \\ \quad + C_{xr}\left(\frac{\cos\delta_r\dot{\theta}_r R - \dot{x}}{\dot{x}}\right)\sin\delta_r - C_{yr}\left(\tan^{-1}\left(\frac{\dot{y}-b\dot{\psi}}{\dot{x}}\right) - \delta_r\right)\cos\delta_r - M\dot{x}\dot{\psi} \end{array}\right] \tag{2}$$

$$\ddot{\psi} = \frac{1}{I}\left[\begin{array}{l} a\left(C_{xf}\left(\frac{\cos\delta_f\dot{\theta}_f R - \dot{x}}{\dot{x}}\right)\sin\delta_f - C_{yf}\left(\tan^{-1}\left(\frac{\dot{y}+a\dot{\psi}}{\dot{x}}\right) - \delta_f\right)\cos\delta_f\right) \\ \quad + b\left(C_{yr}\left(\tan^{-1}\left(\frac{\dot{y}-b\dot{\psi}}{\dot{x}}\right) - \delta_r\right)\cos\delta_r - C_{xr}\left(\frac{\cos\delta_r\dot{\theta}_r R - \dot{x}}{\dot{x}}\right)\sin\delta_r\right) \end{array}\right] \tag{3}$$

$$\ddot{\theta}_f = \frac{1}{I_{wf}}\left[T - RC_{xf}\left(\frac{\cos\delta_f \dot{\theta}_f R - \dot{x}}{\dot{x}}\right)\right] \tag{4}$$

$$\ddot{\theta}_r = \frac{1}{I_{wr}}\left[\frac{\cos\delta_r R^2 C_{xr}\dot{\theta}_r - \dot{x}}{\dot{x}}\right] \tag{5}$$

The parameters for the model are based on those of a large front wheel drive car [10]. In addition, the tyre traction forces are obtained using either a linear or non-linear tyre model (Fiala [11]). These parameters are shown in tables 1 and 2. From equation 1, a simple aerodynamic drag force term has been added with the inputs being the front wheel drive torque, T, and the front and rear steer angles, δ_f, δ_r respectively.

M	Mass	1150	kg
I	Yaw inertia	1850	kgm^2
I_{wf}, I_{wr}	Front/rear spin inertia	12, 1.2	kgm^2
a, b	Dimension of CoG to front/rear axles	1.064, 1.596	m
C_x^d	Drag coefficient	0.36	---
A_x	Frontal area	2.06	m^2
ρ	Air density	1.29	kgm^{-3}
R	Rolling radius of wheels	0.2	m

Table 1: Parameter values for vehicle model

C_{xf}, C_{xr}	Longitudinal tyre stiffness	200, 200	kN/κ
C_{yf}, C_{yr}	Lateral tyre stiffness	140, 120	kN/α
μ_0, μ_1	Static/dynamic Coefficient of friction	0.4, 0.2	---
F_{zf}, F_{zr}	Static front/rear wheel reactions	---	kN
α	Lateral slip	---	radians
κ	Longitudinal slip	---	---

Table 2: Tyre model parameter

Later, the equations of motion are linearised to give Jacobean matrices for implementation in the KF design.

KALMAN FILTER DESIGN

As mentioned earlier, the EKF is used as a state estimator for the measured state variables for a plant subject to measurement noise and plant disturbances. The EKF design involves 2 stages: namely those of correction and prediction [12]. In the first stage, the corrector optimally estimates the state variables such that the covariance between the measured and the estimated states is minimised. The estimated states are then input to the predictor stage which generates new state estimates that are fed back to the corrector stage at the next time step.

Due to the non-linear nature of the plant an EKF is required [12, 13]. There are a variety of schemes that can be implemented but in general an EKF involves repeated linearisation of the plant model as the plant operating point changes. The EKF design and associated linearisation process can be done off-line and then the appropriate KF can be selected from a bank of stored designs. Alternatively, an EKF can be implemented on-line by generating the Jacobian matrices of linearisation at each time step. The estimated state outputs can then be used by a controller/compensator. The corrector stage of the KF contains a linearised model of the plant in terms of the Jacobian matrices, where as the predictor part of the KF contains a non-linear model of the plant.

Another consideration in the design of the KF is the nature of the noise and plant disturbances. These can be treated as stationary or time varying stochastic processes and include cross coupling between the noise signals. If the stochastic properties of the noise and disturbances are non-stationary then the designed KF will be recursive [14].

IMPLEMENTATION

In this paper, the on-line evaluation of the Jacobian matrices approach for the EKF design is performed. In the linearisation process therefore simplifying assumptions are made for the vehicle model, such as using the small angle theorem and linear tyre models. Additionally, for the vehicle model in the predictor phase of the KF design the numerical integrators provided by Simulink® are used to solve the equations of motion. Alternatively bespoke numerical integration schemes, such as Euler or Runge-Kutta could have been coded.

REAR WHEEL STEER

The estimated state outputs from the KF are input to a RWS (rear wheel steer) controller .The RWS algorithm used in this work is based on work by Senger [15]. In this approach the RWS angle is generated using a transfer function derived with respect to the front steer angle and the forward speed of the vehicle:

$$\begin{cases} \delta_r = K_\delta \delta_f \\ K_\delta = \dfrac{-b + \left(Ma / [C_r l] \right) \dot{x}^2}{a + \left(Mb / [C_f l] \right) \dot{x}^2} \\ l = a + b \end{cases} \tag{6}$$

The objective of the algorithm is to minimise the side-slip angle of the vehicle by minimising the lateral motion in comparison to the yaw motion for a given turn manoeuvre.

SIMULATION ENVIRONMENT

The transient simulation in this paper is based on a test case manoeuvre where the vehicle accelerates up to a steady speed of 25ms^{-1} and then initiates a right hand turn. The longitudinal and lateral driver models are based on simple proportional control of the forward speed and yaw rate of the vehicle with the control outputs drive torque and front steer angle. The plant model and the model in the predictor stage of the EKF design are configured to use either a linear or non-linear (Fiala) tyre model depending on a settable program switch. In order to simulate the effect of plant disturbances, the longitudinal traction force is factored by 0.9 and 1.1 between 15 to 20 seconds of the simulation duration in order to simulate the effect of climbing and descending a gradient.

In this paper four simulation environments are created. Each environment can be separated into two sets of pairs of individual simulation runs, one set runs using the rear wheel steer algorithm while the other does not. In each set, one simulation is run using direct full state measurement and one using indirect full state measurements, see Table 3. For the two simulations with indirect full state measurement, the lateral acceleration, yaw rate and front and rear spin rates are measured in order to derive the other states. Each simulation environment A to D is run four times for each of the tyre model permutations (subsets) given in Table 4. All simulation subsets listed in Table 4 use time varying noise properties, except subset 4 which uses stationary noise and disturbance properties. For the EKF design, the noise and plant disturbance covariance values are listed in Table 5.

Simulation environment	RWS	Direct/Indirect
A	With	Full
B	With	Partial
C	Without	Full
D	Without	Partial

Table 3: The four simulation environment

Subset	Plant	Predictor
1	Linear	Linear
2	Fiala	Fiala
3	Fiala	Linear
4	Fiala	Fiala

Table 4: Tyre model used with environments A to D

	Variance		Variance
x	0.1	$\dot{x}$	0.003162
y	0.1	$\dot{y}$	0.3162
ψ	0.3162	$\dot{\psi}$	0.1
θ_f	0.1	$\dot{\theta}_f$	3.162
θ_r	0.1	$\dot{\theta}_r$	3.162
$T(torque)$	10	δ_f	1.00E-04
		δ_r	1.00E-04

Table 5: Disturbance and measurement covariance values used for the KF design

RESULTS

Figure 2 shows a typical time response graph for the longitudinal and lateral vehicle velocities. Notice that the EKF estimates are tracking the plant response well whilst attenuating the noise significantly. This good tracking performance is a result of careful 'tuning' of the $\dot{x}$ measurement covariance for the EKF design. Tables 6 and 7 show the measurement and estimate state variable covariance values for simulations corresponding to environments A to D for tyre permutations 1 and 3 (see tables 3 and 4).

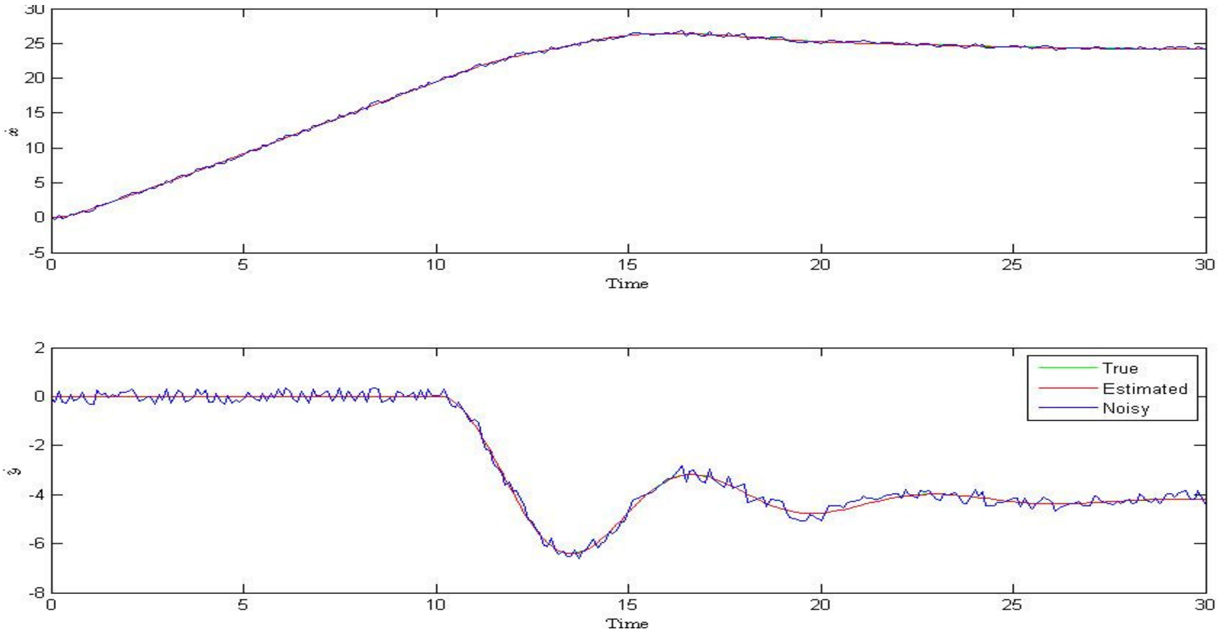

Figure 2: Longitudinal and Lateral velocity responses showing true, measured and estimated responses.

From table 6 it can be seen that the estimation performance in terms of error covariance is superior for the EKF design using direct full state measurement in comparison to indirect full state measurement. Table 7 shows the results for subset 3 corresponding to differing tyre models being used in the simulation. These results show that the EKF is not able to track the plant response accurately when the plant and predictor use different tyre models. This demonstrates that for the EKF to be robust, and therefore practically useful, it must be able to either incorporate an accurate vehicle model or use a vehicle model in conjunction with a parameter identification algorithm for those parameters that have the greatest effect on the estimation performance. From these results, if the EKF is used in its current form therefore, as long as the same tyre model is used in the plant and the predictor, the estimated error covariance will be lower than the measured error covariance.

	No rear steer				Rear steer			
	Full state measurement (GPS)		Three state measurement (estimation)		Full state measurement (GPS)		Three state measurement (estimation)	
	Measurement error	Estimate error	Measurement error	Estimate error	Measurement error	Estimate error	Measurement error	Estimate error
$\dot{x}$	0.0376	9.70e-4	0.1787	0.0012	0.0376	7.03e-4	0.1778	8.86e-4
$\dot{y}$	0.0376	2.53e-5	0.0082	2.93e-5	0.0376	1.79e-5	0.0082	2.02e-5
$\dot{\psi}$	3.76e-5	2.96e-8	3.76e-5	2.84e-8	3.76e-5	1.94e-8	3.76e-5	1.86e-8
$\dot{\theta}_f$	3.7636	0.3259	3.7636	0.3243	3.7636	0.2303	3.7636	0.2311
$\dot{\theta}_r$	3.7599	0.0260	3.7599	0.0312	3.7599	0.0191	3.7599	0.0236

Table 6: Error covariance values for subset 1

	No rear steer				Rear steer			
	Full state measurement (GPS)		Three state measurement (estimation)		Full state measurement (GPS)		Three state measurement (estimation)	
	Measurement error	Estimate error	Measurement error	Estimate error	Measurement error	Estimate error	Measurement error	Estimate error
$\dot{x}$	0.0376	11.93	0.1712	11.97	0.038	8.402	0.1717	8.4351
$\dot{y}$	0.0376	1.2222	0.0082	1.2188	0.0376	0.5971	0.0082	0.5960
$\dot{\psi}$	3.76e-5	4.90e-4	3.76e-5	4.89e-4	3.761e-5	1.81e-4	3.76e-5	1.81e-4
$\dot{\theta}_f$	3.7635	1.84e+3	3.7635	1.84e+3	3.7635	2.02e+3	3.7635	2.02e+3
$\dot{\theta}_r$	3.7598	295.4	3.76	296.36	3.7598	208.6	3.7598	209.4

Table 7: Error covariance values for subset 3

CONCLUSION

In conclusion the work reported on in this paper covers the development of an EKF simulation based on on-line KF design for a non-linear five degree of freedom transient vehicle plant model. In order to investigate the EKF design robustness the same basic simulation and test case manoeuvre was repeatedly run for both stationary and time varying measurement and disturbance noise, including gradient induced plant disturbance. In addition, the performance of the estimator in terms of robustness for the case of plant uncertainty was simulated by the inclusion of various permutations of linear and non-linear tyre model in the plant and KF design. In order to investigate the effect of the use of direct and indirect full state measurement both observed indirect full state measurement utilizing sensor fusion and direct full state measurement assuming the use of GPS compensated INS sensors were simulated. In summary it was found that smaller estimation errors were obtained when the plant was subject to stationary noise and that direct full state measurement yielded superior state estimation error performance in comparison to indirect full state measurement utilising sensor fusion. Although, the exact measure of this identified direct full state measurement performance advantage cannot be quantified without further study, the principle was clearly demonstrated.

However, a major finding from this work was the robustness of the EKF estimation performance. It was clearly shown that with an accurate vehicle model incorporated into the EKF design, as simulated by a linear or non-linear tyre model in both the EKF and the plant, the EKF successfully tracked the plant state variable response and produced state estimate with smaller error covariance than the state measurement errors. Conversely, the state estimate error covariances are significant higher than that of the state measurements when the EKF model were inaccurately modelled and differed from that of the plant. Therefore, the requirement for a robust EKF algorithm to include adaptability, i.e. parameter identification, was clearly demonstrated. It was suggested that the parameters of primary importance in terms of parameter identification were the tyre characteristics, including road surface and friction properties. Other parameters requiring identification by an adaptive algorithm include vehicle mass and inertia properties.

FUTURE WORK

It is clear from the work reported on in this paper that for the EKF to be practically useful, the design of EKF has to be adaptable to the changing parameters. Similar findings are given for an automotive application by Ramsbottom et al [13]. Broadly speaking, in the literature there appears to be three ways in which state estimators and observers are made adaptive (parameter identification functionality). These methods (not all references are specific to vehicle dynamic systems) are the dual KF approach [16], the banked iterative KF approach [17,18] and the GPS compensated approach using one or more KFs [19,20,21]. This latter approach is most relevant to the future work planned for this research, and will be commented on further shortly.

The dual KF approach referred to in reference [16] refers to the KF being used to estimate both the states and the parameters where the state vector is extended and partitioned so that one of the state vector partitions contains the estimated parameters, with the other partition containing the state values as normal. However, the increased size of the state vector incurs a numerical efficiency penalty when matrix operations such as inversion and multiplication are implemented when evaluating the Jacobian matrix during plant model linearisation. However, the approach is widely reported in the literature. The banked approach to adaptive KF design [17,18] refers to having an array of off-line evaluated KF designs, each with differing parameter values, so that an appropriate algorithm, such as a genetic algorithm, can be used to weigh the state estimates from the array of off-line designed KFs to achieve optimum adaptive state estimation. Again, the draw back with this approach is the potential numerical efficiency penalty it incurs. The requirement to implement the adaptive KF design in real time is a major consideration in the choice and success of the adaptive approach.

As previously mentioned, of all the work commented on in the literature the most relevant is given by references [19,20,21] where Bevly et al develop a research theme based on the use of GPS compensated inertial sensors to improve state estimation. Specifically, dual GPS receivers are used in conjunction with kinematic KFs (non-model based) to estimate wheel slip values at a sufficiently high update rate to pass to a model based KF in order to identify the tyre cornering stiffness properties. These identified tyre parameters are then used with another KF to more accurately estimate the key vehicle states. In future work it is highly likely that this approach will be investigated in order to make the EKF so far developed adaptable.

Alternatives do exist to the problem of measuring velocity over the ground states other than GPS compensated INS sensors, namely state observation using data fusion together with error compensation schemes, and direct measurement using optical sensors and radar based transducers. However, these techniques are either prohibitively expensive to reliably implement in normal operating conditions or have inherent performance limitations that impose restrictions on what can be done with the signals they generate in terms of control system performance. In principle therefore, GPS compensated INS sensors offer a potentially cheap and effective means of estimating velocity over the ground states. It is a matter of on-going research to address issues such as GPS down time (and implied INS dead reckoning) and 'canyoning' induced errors together with the expected overall system performance issues incurred when using automotive standard INS and GNS (global navigation sensors) sensors. Additionally, one of the aims of this paper is to highlight the need for further parameter identification research as well as state estimation development before such systems can be made truly practical and useful.

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