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CONTROLLER DESIGN TO COMPENSATE NETWORK-INDUCED DELAYS USING A TIME DELAY MODEL FOR NETWORKED CONTROL SYSTEMS

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ABSTRACT - The employment of networked control systems (NCSs), where control systems are interconnected by communication networks, is a major trend in modern industrial control applications. The network-induced delays are unavoidable in NCSs, and affect system performance significantly. In this paper, an LQR optimal controller is designed based on a time delay model of an NCS in order to compensate for such time delays. The control performance of the NCS is validated through a case study.

TECHICAL PAPER

INTRODUCTION

A networked control system (NCS) is a distributed real-time control system where sensors, controllers, and actuators are interconnected via a network. The NCS provides many advantages, such as the low cost of installation, ease of maintenance, higher reliability, and greater flexibility. The NCSs have already been used in many industrial control applications, such as the automotive and aircraft industries. However, the development of the NCS is difficult in that the network-induced delays are inevitable because of the real-time characteristics and distributed architecture of the NCS. The time delays in the NCS may deteriorate the system performance and cause the system instability. Therefore, it is necessary to design a controller which can compensate for the time delays and improve the control performance of the NCS.

The research of the time delay models for NCS controllers has been done since 1990s [1][2][3]. Most previous studies assume that all time delays are smaller than a sampling period of the NCS, and an actuator responds immediately to a control command, or sensor-to-controller delays and controller-to-actuator delays are regarded as lumped delays. The time delay model of the NCS used in this paper considers the effects of network-induced delays which are time-varying and may be longer than one sampling period [4][5]. The time delay model can make the analysis of temporal behaviors of NCSs more accurately than previously proposed models. In order to guarantee the control performance even though there are the network-induced delays in an NCS, an LQR controller is designed by using the time delay model for delay compensation. In addition, a controller design method is proposed in order to reduce the processing load of the controller for the implementation.

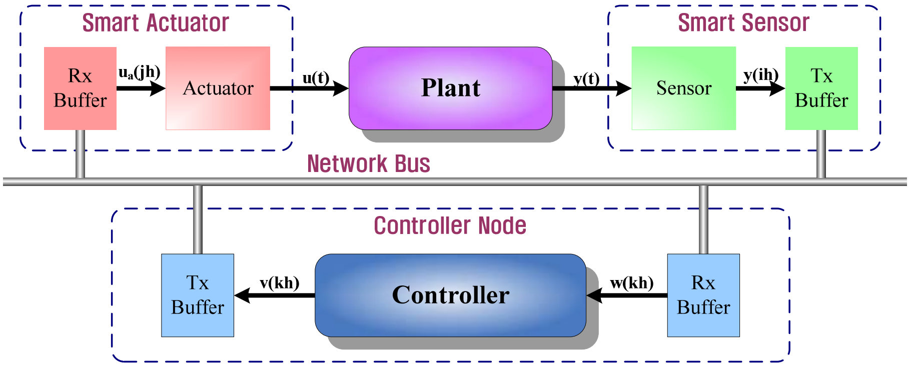

Figure 1. Structure of a SISO NCS

TIME DELAY MODEL[4][5]

The NCS in Figure 1 is a periodic single input and a single output (SISO) system. A periodic NCS is a type of the NCSs that all the nodes are activated with the same sampling period. We utilize the periodic time delay model for a periodic NCS referred to [4][5].

The continuous time, linear time-invariant plant is described in the standard state-variable form as follows:

$$\dot{\mathbf{x}}(t) = \mathbf{A}_\mathrm{p}\mathbf{x}(t) + \mathbf{B}_\mathrm{p}u(t)$$
$$y(t) = \mathbf{C}_\mathrm{p}\mathbf{x}(t) \quad (1)$$

where, $\mathbf{x}(t) \in \mathbf{R}^q$, $\mathbf{A}_\mathrm{p} \in \mathbf{R}^{q\times q}$, $\mathbf{B}_\mathrm{p} \in \mathbf{R}^{q\times p}$, and $\mathbf{C}_\mathrm{p} \in \mathbf{R}^{n\times q}$.

The solution of Equation (1) at time t, where $t_0 \le t \le t_f$, is given by

$$\mathbf{x}(t) = e^{\mathbf{A}(t_f - t_0)}\mathbf{x}(t_0) + \int_{t_0}^{t_f} e^{\mathbf{A}(t_f-\tau)}\mathbf{B}u(\tau)d\tau . \quad (2)$$

Between the sampling instants defined as kh of the controller, i.e. $t_f=(k+1)h$, and $t_0=kh$, Equation (2) can be changed as follows:

$$\mathbf{x}((k+1)h) = \mathbf{\Phi}(h)\mathbf{x}(kh) + \int_{kh}^{(k+1)h} e^{\mathbf{A}((k+1)h-\tau)}\mathbf{B}u(\tau)d\tau \quad (3)$$

where, $\mathbf{\Phi}(h) = e^{\mathbf{A}h}$.

In Figure 1, the plant input, $u(t)$, is influenced by the execution delay of actuation, $\tau_{exe,act}$, after the actuator input, $u_a(jh)$ is sampled at the actuator. The controller-to-actuator delay, τ_{ca}, affects $u_a(jh)$ after the controller output, $v(k)$, is sent from the controller node. Therefore, $u(\tau)$ in Equation (3) can be replaced with $v(k)$ and delay factors. Finally, the time delay model of the plant can be represented as Equation (4)

$$\mathbf{x}(k+1) = \mathbf{\Phi}(h)\mathbf{x}(k) + \sum_{m=0}^{a_{act}+a_{ca}+1} \mathbf{f}_{m,k} v(k-m) \quad (4)$$

where $\mathbf{f}_{m,k} \in \mathbf{R}^{n\times 1}$ is the coefficient vector containing the delay information. a_{act}, and a_{ca} stand for maximum allowable delay bounds(MADBs) of actuation and controller-to-actuator delays respectively.

A linear time-invariant feedback control law in the discrete time is represented as follows:

$$v(k) = -\mathbf{K}_\mathrm{c} w(k) \quad (5)$$

where $\mathbf{K}_\mathrm{c} \in \mathbf{R}^{n\times n}$ is the control gain.

In Equation (5), the controller output, $v(k)$, is calculated with the controller input, $w(kh)$, and the control gain, $\mathbf{K}_\mathrm{c}$. $w(kh)$ is affected by the sensor-to-controller delay, τ_{sc}, after the plant output, $y(ih)$, is sent from the sensor node. From Equations (1) and (3), $w(k)$ can be expressed by the $x(k)$ and $v(k)$ with the delay factors. $v(k)$ is represented as Equation (6).

$$v(k) = -\mathbf{K_c}\left(\sum_{n=1}^{a_{sc}+1}\mathbf{g}_{n,k}\mathbf{x}(k-n) + \sum_{n=1}^{a_{sc}+a_{ca}+a_{act}+2} r_{n,k}v(k-n)\right) \quad (6)$$

where $\mathbf{g}_{n,k}$ and $r_{n,k}$ are coefficient vectors. a_{sc} is MADB of sensor-to-controller delay. All the time parameters and the coefficients in Equations (4) and (6) include the all information of the time delays in an NCS and are specifically defined in [4] and [5].

If we assume a state-feedback controller, the controller input, $w(kh)$, is the delayed value of the plant output, $\mathbf{x}(i\text{-}p)h$, which is sensed by the sensor node before ph. For compensation of the network-induced delays, $\mathbf{x}(kh)$ is calculated by using the time delay model for $[(k\text{-}p)h\text{-}\Delta_{sc}, kh)$. $\mathbf{x}(kh)$ is used as $w(kh)$ and represented as Equation (7).

$$\begin{aligned}\mathbf{x}(kh) &= e^{\mathbf{A}\tau_{sc}(k)}x(t_0) + \int_{t_0}^{t_f} e^{\mathbf{A}(t_f-\tau)}\mathbf{B}u(\tau)d\tau = \Phi_s(\tau_{sc,k})\mathbf{x}((i-p)h) + \int_{(k-p)h-\Delta_{sc}}^{kh} e^{\mathbf{A}(kh-\tau)}\mathbf{B}u(\tau)d\tau \\ &= \Phi_s(\tau_{sc,k})\mathbf{x}((i-p)h) + \sum_{n=1}^{p}\sum_{m=0}^{a_{act}+a_{ca}+1}\mathbf{f}_{m,k-n}v(k-n-m) + \sum_{m=1}^{a_{act}+1}\mathbf{\Gamma}_{m,k-p}^{\Delta_{sc}}\sum_{n=0}^{a_{ca}}\beta_{n,k-p-1}v(k-p-1-n)\end{aligned} \quad (7)$$

where, $\Phi_s(\tau_{sc,k}) = e^{\mathbf{A}\tau_{sc,k}}$. $\mathbf{\Gamma}_{m,k-p}^{\Delta_{sc}}$ is an integrated value of the plant ranging from $(i\text{-}p)h$ to $(k\text{-}p)h$. $\beta_{n,k-p-1}$ is the coefficient for $v(k\text{-}p\text{-}1\text{-}n)$ [4].

The time delay model considers the time-varying delays which may be longer than a sampling period, h. In addition, since the sensor-to-controller and the controller-to-actuator delays are separately considered, the time delay model can be easily extended to other NCSs having different network structures. Because of the advantages, the time delay model is employed in order to design a controller.

CONTROLLER DESIGN

A number of control methodologies can be considered to guarantee the control performance. Some control methodologies, such as PID control or pole placement design, do not consider the system uncertainties. A controller is needed to guarantee system stability even if there are time delays in the NCS. In this paper, an LQR controller is designed because the minimum bounds on the robustness of the LQR are guaranteed by manipulating the Riccati equation [6].

A plant model in Equation (1) should be modified to consider the effects of network-induced delays. In Equation (4), the plant input is dependent on $v(k)$ and the time-varying delays. In order to take into account the time delays, a new augmented state variable, $\mathbf{z}(k)$, is defined as Equation (8), which consist of the plant state and the all delayed controller outputs influencing the plant during one sampling period. The augmented state variable with an appropriate dimension is used to design an LQR controller.

$$\mathbf{z}(k) = [\mathbf{x}(k)^{\mathbf{T}}, v(k-1)^{\mathbf{T}}, \ldots, v(k-a_{act}-a_{ca}-1)^{\mathbf{T}}] \in \mathbf{R}^{n+a_{act}+a_{ca}\times n+a_{act}+a_{ca}}. \quad (8)$$

The new state-space equation is represented as Equation (9). The state-space equation is time-variant because of the time-varying delays in the NCS.

$$\mathbf{z}(k+1)=\begin{bmatrix} \mathbf{\Phi} & \mathbf{f}_{1,k} \cdots \mathbf{f}_{a_{act}+a_{ca},k} & \mathbf{f}_{a_{act}+a_{ca}+1,k} \\ \mathbf{0} & 0 \;\cdots\; 0 & 0 \\ \mathbf{0} & 1 \;\cdots\; 0 & 0 \\ \vdots & \vdots \;\ddots\; \vdots & \vdots \\ \mathbf{0} & 0 \;\cdots\; 1 & 0 \end{bmatrix}\mathbf{z}(k)+\begin{bmatrix} \mathbf{f}_{0,k} \\ 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}v(k)=\mathbf{A_z}\mathbf{z}(k)+\mathbf{B_z}v(k)\,. \tag{9}$$

The performance index and the weighting factors for the LQR controller design is defined as follows:

$$J_c(t)=\sum_{k=0}^{N}[\mathbf{x}^T(k)\mathbf{Q}_s\mathbf{x}(k)+\sum_{i=0}^{a_{ca}+a_{act}+1} v(k-i)^T\mathbf{R}_i v(k-i)]=\sum_{k=0}^{N}\mathbf{z}^T(k)\mathbf{Q}\mathbf{z}(k)+v(k)^T\mathbf{R}v(k)\,, \tag{10}$$

and $\mathbf{Q}=\begin{bmatrix} \mathbf{Q}_s & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & R_1 & \cdots & 0 \\ & \vdots & \ddots & \vdots \\ \mathbf{0} & 0 & \cdots & R_{a_{ca}+a_{act}+1} \end{bmatrix}$, $R=R_0$.

The control gain obtained from Equation (10) is expressed as $\mathbf{K_{ncs}}$ and the control law $v(k)$ in Equation (5) can be rewritten as Equation (11)

$$\begin{aligned} v_c(k) &= -\mathbf{K_{ncs}}z(k) \\ &= -\mathbf{K_c}\mathbf{x}(k)-\sum_{m=0}^{a_{act}+a_{ca}+1} K_v^m v_c(k-m) \end{aligned} \tag{11}$$

where, $\mathbf{K_{ncs}}=[\mathbf{K_c}\ K_v^1\ K_v^2 \cdots K_v^{a_{act}+a_{ca}+1}]$.

The control law, $v_c(k)$, in Equation (11) is used in the replacement of $v(k)$ in Equation (4).

CASE STUDY

In the case study, an inverted pendulum in Figure 2 is considered. The inverted pendulum system is assumed to be a periodic NCS. All the nodes are periodically activated with the same sampling period but asynchronously activated. In this system, the time delays are time-varying and may have longer delays than a sampling period.

LQR Controller Design

The inverted pendulum model is represented as Equation (12)

$$\ddot{\theta}=w_o^2\theta+\frac{w_o^2}{g}u \tag{12}$$

where $w_o=\sqrt{\dfrac{g}{l}}$ is the natural frequency of the pendulum with l and $\dot{\theta}$ is the angular velocity (rad/sec) of the pendulum.

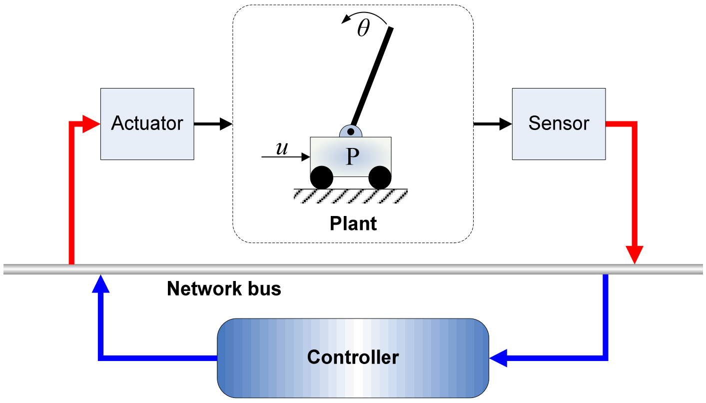

Figure 2. Inverted pendulum using an NCS

The state variable, $\mathbf{x} = [x_1 \;\; x_2]^T = [\theta \;\; \dot{\theta}]^T$, is defined. The state space model of the inverted pendulum can be rewritten as Equation (13) where state space models are observable.

$$\dot{\mathbf{x}}(t) = \begin{bmatrix} 0 & 1 \\ w_0^2 & 0 \end{bmatrix} \mathbf{x}(t) + \begin{bmatrix} 0 \\ \frac{w_0^2}{g} \end{bmatrix} u(t), \quad \mathbf{x}(0) = \begin{bmatrix} 0.2 \\ 0 \end{bmatrix}. \tag{13}$$

The assumptions for the simulation are as follows:

1) $l = 0.3m$
2) $h = 20ms$
3) $\Delta_{sc}, \Delta_{ca} = 0.6ms$
4) $a_{sc} = 2$, $a_{ca} = 2$, $a_{exe} = 1$
5) $\tau_{sc\_t} \le a_{sc} h$, $\tau_{ca\_t} \le a_{ca} h$, $\tau_{exe,act} \le a_{act} h$

Travel times, τ_{sc_t}, τ_{ca_t}, and τ_{act}, are random values satisfying the assumption 5). The time skews, Δ_{sc} and Δ_{ca}, are the time difference between the sampling instants of the nodes. The terms in the above assumptions are defined in [4][5].

If there is no delay in the NCS, the performance index and weighting factors are as follows:

$$J_c(t) = \sum_{k=0}^{N} \mathbf{x}^T(k)\mathbf{Q}\mathbf{x}(k) + v(k)^T \mathbf{R} v(k) \tag{14}$$

where, $\mathbf{Q} = \begin{bmatrix} 100 & 0 \\ 0 & 10 \end{bmatrix}$, and $\mathbf{R}$=1. (15)

The optimal control gain is obtained as Equation (16):

$$\mathbf{K_c} = [23.8014 \quad 4.9276]. \tag{16}$$

The simulation result shows that the controller performance of the NCS without delay is satisfactory as the dotted line in Figure 3. However, when the controller designed as Equation (16) is applied to the NCS which have network-induced delays, it is shown that the control performance is degraded, and the angular poison and the angular velocity of the inverted pendulum oscillate because the controller does not compensate for the time delays in Figure 3.

In order to compensate for the time delays of the NCS, an LQR controller using the time delay model is designed. From Equation (10), the performance index is expressed as follows:

$$J_c(t) = \sum_{k=0}^{N} \mathbf{z}^T(k)\mathbf{Q}\mathbf{z}(k) + v(k)^T \mathbf{R} v(k) . \qquad (17)$$

and the weighting factors **Q** and **R** are defined as Equation (18).

$$\mathbf{Q} = \mathbf{diag}(100\ 10\ 0.8\ 0.5\ 0.3\ 0.1), \ \mathbf{R} = 1. \qquad (18)$$

As the simulation result, the controller compensating for the time delays improves the control performance. In this case, the system matrices, $\mathbf{A_z}$ and $\mathbf{B_z}$, in Equation (9) are time-varying every sampling period because of the time delays. The control gain should be calculated every sampling period. However, calculating the control gains in this way can burden the processing load of the controller, and it is inappropriate to implement the NCS.

In order to reduce the processing load of the controller, a controller with a fixed control gain is designed by considering MADBs. The maximum time delays are determined by assumptions 4) and 5). Therefore, the fixed control gain for the inverted pendulum is calculated by using Equations (17) and (18) as follows:

$$\mathbf{K_{ncs}} = [29.6063 \quad 5.3665 \quad 0.2640 \quad 0.2940 \quad 0.3278 \quad 0.1056]. \qquad (19)$$

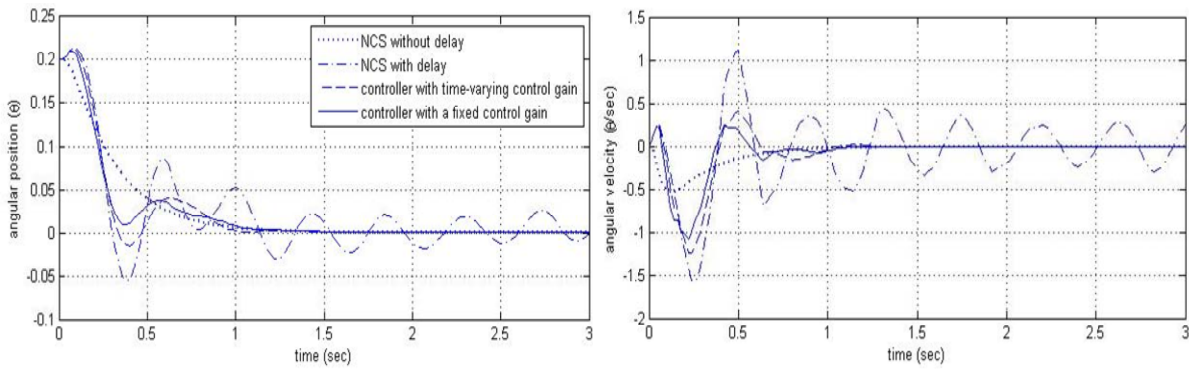

Figure 3. Transient response of the inverted pendulum

In Figure 3, it is shown that the control performances of the NCS controllers with the time-varying control gain and the fixed control gain. For the controller with the fixed controller gain based on MADBs, the response converges a little slowly. However, the control performance is better than the controller with time-varying control gains. Moreover, the calculation processing is remarkably reduced. Therefore, the controller designed by using MADBs is feasible to implement an NCS.

CONCLUSION

The time delay model of an NCS used in this research is more realistic than previously proposed time delay models in that it considers the time delays which are time-varying and may be longer than a sampling period. Since the time delay model considers all the temporal behaviors of the NCS, such as sensor-to-controller delays, controller-to-actuator delays, and the computation delays cause by task execution, we could precisely examine the impact of network-induced delays on the system performance.

In order to compensate for the network-induced delays, the LQR controller was designed by using the time delay model [4][5]. The LQR controller needs to updates control law every sampling period to compensate for time-varying delays. However, it is inappropriate for the implementation. If MADBs of an NCS can be known, the controller can be designed based on the MADBs. We could investigate the feasibility of the implementation of the controller based on the MADBs because the calculation processing of the controller is reduced with the satisfactory control performance.

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