TECHNICAL PAPER FOR STUDENTS AND YOUNG ENGINEERS

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TITLE:

POWERTRAIN VIBRATION: MODELLING, SIMULATION AND TESTING

Topic:

- ○ FUTURE AUTOMOTIVE TECHNOLOGY
- ○ INTELLIGENT TRANSPORTATION SYSTEMS
- ● USER FRIENDLY AUTOMOBILE
- ○ ADVANCED PRODUCTION AND LOGISTICS
- ○ VEHICLES & THE ENVIRONMENT

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Abstract:

Automotive engineers use modelling, simulation and testing to reduce powertrain vibration. This is especially important in passenger vehicles for comfortable ride. This paper provides an outline for methodologies behind this work, from developing mathematical models, to simulation and experimental verification. Modelling is demonstrated using torsional finite elements for direct, branched and grounded (boundary) connections and geared connections. The use of these elements for developing dynamic models of various passenger vehicle powertrains is illustrated and discussed. A six degree of freedom dynamic model for clutch engagement is used to demonstrate numerical simulations for transient vibration from clutch engagement stick-slip and gear backlash. A powertrain test rig that is used for measuring torsional vibration is presented. Test methods used to verify models and analysis are discussed for free vibration, steady state and transient response.

Place / Date:

Sydney, 10/02/04

1.0 INTRODUCTION

Powertrain systems can be investigated for vibration characteristics via *dynamic modelling*, *analytical and numerical analysis* and *testing*. These investigations lead to a better understanding of the causes and effects of vibration and thus allow improvement in design of passenger cars.

Powertrains can be *modelled* as torsional elastic systems. Basic models start with linear-lumped spring-mass systems of equations of motion and improve with additional detail for the dynamics of various components. These models can be used to estimate the natural frequencies of the system. They can be described with torsional finite elements and assembled into dynamic systems [1]-[3]. The models can be used to reduce resonance by designing system natural frequencies away from engine harmonics. Commonly, automatic transmission equipped powertrains have torque converter lock-up damper clutches with high hysteresis and single or multistage stiffness, in designing these clutches, stiffness and hysteresis parameter sensitivity analysis can be used to reduce resonance [4]. Often this is presented as Campbell plot [4] - relating engine harmonics and speed to the spring system natural frequencies. Stiffness variation in other shafting can be similarly analysed [5]. Some models use distributed mass elements for long shafts with significant mass, such as the propeller or centre drive shaft [6], these elements are integrated as a lumped-distributed mass system.

Numerical solutions for dynamic models can be found for transient vibration from tip-in engine torque, gear shifts, etc. [2], [4]-[9]. The most commonly investigated transient occurs as the lowest global mode of the torsional system, it is called driveline shuffle. It is largely dependent on the engine/flywheel inertia, drive shaft stiffness and overall gear ratio. The mode's frequency increases from first gear to top gear. This is as it is a global mode of vibration. Local modes in the powertrain that are away from the transmission are not affected by its gearing.

Models can be improved with addition of non-linear phenomena and solutions found *numerically* for transient dynamics. *Judder* is a friction induced vibration between masses with sliding contact. Judder in automotive clutches has been a focus of previous research in both manual and automatic transmissions. It has long been attributed to an *increasing* dynamic friction coefficient with *decreasing* slipping velocity, dubbed as a negative friction gradient. Judder is known to be reduced by system damping. The effect of the damping depends on its location being within the excited mode. Wet clutch and slipping clutch design calls for the correct combination of friction material and automatic transmission fluid to produce a continuously positive sloped friction curve. This helps prevent driveline torsional oscillations excited by clutch judder. [7]

Stick-slip is the non-linear intermittent sliding and stichon (sticking) at a contact surface. It is a phenomenon with dependence on friction characteristics, system dynamics and external tangential and normal forcing. Most studies concentrate on systems with the sliding interface between one moving and one stationary surface or between one moving surface and a substrate moving at a predetermined velocity. A great deal of analytical work has been published for various systems [10]. In automotive clutches the sliding interface is between two masses with undetermined velocities in a non-linear non-autonomous system, thus investigation using numerical methods is appropriate [7].

Geared systems require clearance between mating gears for smooth operation. The clearance is termed *lash* and the mating gears must separate across the lash when their relative directions of rotation change. The mating gears can be modelled with a mesh stiffness which is non-linear [2], [8] and [11]. It is set as zero across the lash zone. 'Clonk' is a term used in the automotive industry for the noise produced from impacting gears and splines during torque reversal. Transient dynamics from engine tip/in, gear shifts, etc. can produce a torque reversal (shuffle) thereby inducing clonk. Research [9] has shown the relationship between clonk and shuffle.

Powertrain testing can be used to verify the models and analysis. Testing can be on-vehicle and with various rigs. Some test rigs have the entire vehicle powertrain on mounts with added vehicle load and equivalent mass. Instrumentation includes microphones, accelerometers, torque sensors and the like. Typical investigations are for free, steady state and transient response and gear shift calibration. One rig was built that simulated engine harmonics by use of the varying velocity of a Hooke's joint [2]. They compared tests with analysis in which they included stiffness for the input and output clutch shaft, gear teeth and shafts, drive shafts and wheels. A damping ratio of 6% was used for shuffle. Investigation included the torsional phenomena of idle gear rattle, drive rattle, surging and boom. Another rig was built for a light diesel truck powertrain fitted with a manual transmission [6]. They investigated shuffle and clonk finding good comparison between analysis and test results. A similar rig is detailed in [9]. Both rigs were used for frequency analysis by inducing free vibration. The test rig tires were grounded and a vibration response induced from a shock load to the flywheel or torque release. Shuffle frequencies were measured globally while clonk was detected mostly at the final drive and transmission housing.

2.0 MODELLING WITH TORSIONAL FINITE ELEMENTS

Torsional finite simplify powertrain modelling. They represent inertias, their local coordinates and coupling within global dynamic systems. These elements are used to develop a global system of equations of motion via a simple matrix assembly [1], [3]. Model schematics are shown in figure 1 (all figures and tables are in the appendices) for five simple dynamic systems with lumped inertias and connecting damping and stiffness. The five schematics are examples for *direct*, *geared – rigid* and *elastic mesh, branched* and *grounded* systems. Stiffness and damping parameters are torsional except for the geared connection with elastic mesh where the stiffness parameter represents tooth stiffness normal to the plane of contact. For each system the required torsional finite elements are outlined. The derivation for each element type is not provided as it is elementary. The matrices for inertia, stiffness and damping and the local coordinate vectors are given in table 1.

The general finite element types presented can be used for quickly obtaining the equations of motion for large complicated systems. It can be used for lumped inertia torsional systems and is particularly useful for vehicle powertrain applications.

Modelling vehicle powertrain systems fitted *automatic transmissions* can be complicated but the finite element method simplifies the task considerably. It is appropriate for powertrain systems with single stage or multi-stage planetary gear sets. A schematic for such a system is given in figure 2. In this system k_1, k_7 and k_9 (input shaft and tire stiffness) are direct elements and k_6 and k_8 are rigid geared and branched elements (drive shafts and differential gearing). The remaining connections – those in and out of the planetary gear set are described by a custom finite element. The complete derivation for this element is provided in [3]. Briefly, the element is derived from equations of motion for gear components that include the internal forces and external torques and from the constraining acceleration relationships between the components. The element is general and can be modified for each gear state when placed in the surrounding powertrain system. The transmission has many states of operation – first through to fourth gears and torque converter lock-up, with clutches and bands controlling gear shifts and their states defining the motion of the gearset components. Using the torsional finite elements the global system can be quickly assembled for any of these states. The final set of equations includes the complete dynamics of the planetary gear set. This same methodology can be applied to five and six speed automatic transmissions.

Continuously Variable Transmissions (CVT) are the most recent type of transmission to be widely used in vehicle powertrains. Common types are toroidal, v-belt and hydromechanical CVT's. These systems can be even more complicated than automatic transmissions as some have multi-staging and some are used in tandem with planetary gear sets – then requiring clutches and/or brake bands. *The finite element method provides an appropriate tool for the dynamic modelling of these systems*. Figure 3 presents a model for a powertrain fitted with a half toroidal CVT and planetary gear set. There are two clutches, a high velocity clutch (HVC) which connects the toroid direct to the differential and a low velocity clutch (LVC) which connects the toroid to the differential via a single stage planetary gear set. In this system the power can flow either way depending on the clutch engagement. The connection between the LVC and the ring gear (via the sun gear), k_6 and c_6, are modelled as geared elements. Note the gear set is modelled with equivalent ring gear coordinates. The connections from the differential to the wheels, k_9 and c_9, and k_{10} and c_{10}, are modelled as geared and branched elements.

Torque is transferred between the toroids and the roller via a thin film of oil that transiently acts like a solid. This film can be represented with a damping and stiffness. Custom finite elements have been derived to represent this connection. Connections k_2, c_2 and k_3, c_3 are considered as horizontal. With radiuses r_2, r_3 and r_4, torsional stiffness is introduced as (likewise for torsional damping):

$$k_2' = r_3^2 k_2 \quad \text{and} \quad k_3' = r_4^2 k_3 \qquad (1)/(2)$$

The derived elements are given in table 2. Note coordinates 2 and 4 (toriods) have positive rotation clockwise. Coordinate 3 (roller) has positive rotation anti-clockwise, if the signs of the stiffness/damping coefficients in the element are all made positive it will be clockwise. In either case in solution the roller rotates opposite to the toriods. All other connections in the system are direct elements. The global system can be quickly assembled from these elements with a global coordinate vector for either low velocity or high velocity clutch engagement.

3.0 POWERTRAIN MODEL FOR CLUTCH ENGAGEMENT

Figure 4 presents a dynamic model used for investigating clutch engagement and gear lash non-linear dynamics. Six degrees of freedom (DOF) are modelled - engine, clutch drum, clutch hub, differential ring gear, differential pinion gear and tire/vehicle mass. The four stiffness elements are input shaft, propeller shaft, tooth mesh and drive shaft. The system parameters are given in table 4. They are approximate to a passenger car. Transmission gearing is assumed as 1:1, transmission inertia is lumped to the clutch hub. This model is simplified to more clearly illustrate simulation techniques and the causes and effects of these non-lineararites in powertrain systems. The technique can be applied in particular vehicle configurations as discussed in section 2. The finite element inertia, stiffness and damping matrices and local coordinate vectors for this six DOF system are given in table 3 (25)-(28). *They are assembled into global system matrices* (3) *by using local coordinate vectors and the global coordinate vector* (4) *or* (6). When the clutch is disengaged or slipping the system has six DOF (4). When the clutch is engaged the system has five DOF (6), either coordinate 2 or coordinate 3 can represent the engaged clutch inertia. The final equation of motion for the system will have the form:

$$I\ddot{\theta} + C\dot{\theta} + K\theta = T \qquad (3)$$

With global coordinate and torque vectors:

$$\theta = \{\theta_1 \quad \theta_2 \quad \theta_3 \quad \theta_4 \quad \theta_5 \quad \theta_6\} \qquad T = \{T_E \quad T_C \quad -T_C \quad 0 \quad 0 \quad -T_V\} \qquad \text{slipping} \quad (4)/(5)$$

$$\theta = \{\theta_1 \quad \theta_{2/3} \quad \theta_4 \quad \theta_5 \quad \theta_6\} \qquad T = \{T_E \quad 0 \quad 0 \quad 0 \quad -T_V\} \qquad \text{engaged} \quad (6)/(7)$$

4.0 LINEAR AND NON-LINEAR ANALYSIS AND SIMULATIONS

4.1 Free Vibration Analysis

The system can be *linearised* for free vibration analysis by setting the torque vector (5) or (7) to zero and omitting gear lash. For the analysis, the tire/vehicle inertia coordinate can be grounded or free-spinning, it makes little difference as the effective vehicle inertia is so large compared to the rest of the system. If it is free spinning the eigenvalues of the system have a zero value or pair, representing the rigid body motion of the system, the mode shape reflects the gear ratio. The five DOF system (engaged clutch) is fully coupled whereas the six DOF system is uncoupled at the disengaged/slipping clutch. For either configuration there are four natural frequencies and corresponding mode shapes (table 5).

Clutch engagement judder has been investigated for the powertrain with automatic transmission configuration of figure 2 [7]. The clutch torque is represented as a function of clutch geometry, applied pressure and the sliding friction coefficient. The system can be *linearised* for the slipping clutch by assuming a constant applied pressure and defining the friction coefficient as a line function. The clutch torque is then autonomous as it is described by a constant and a function of relative velocity. The magnitude and sign of the relative velocity function is dependent on the gradient of the friction coefficient. It is equivalent to viscous damping. If the gradient is positive/negative the roots of the system are stable/unstable. Other damping elements can stabilise the system.

4.2 Forced Vibration Analysis

Non-linear forced vibration analysis includes:

- Engine torque as a function of engine speed and throttle position.
- Clutch torque as a function of apply pressure and the sliding friction coefficient
- Clutch engagement stick/slip piecewise analysis
- Gear mesh clearance non-linearity

The engine torque can be described an engine characteristic map for a typical four cylinder gasoline engine. The engine torque is a function of the engine speed and the throttle position.

Assuming constant pressure across the surface of the clutch plates, the equation for clutch torque is:

$$T_C = N_s R_m \mu_S F \qquad R_m = \frac{2(r_o^3 - r_i^3)}{3(r_o^2 - r_i^2)} \qquad (8)/(9)$$

Where Ns is the number of friction surfaces, R_m is the clutch mean radius, μ_S the coefficient of sliding friction and F the clutch actuating force. The actuating force is determined from applied pressure which is non-linear and a function of time. The mean clutch radius is determined from the clutch outside (r_o) and inside (r_i) radii:

Clutch Stick-Slip – Friction Non-linearity

When the clutch engages the system changes state as there is one less DOF. If there is not enough holding torque the clutch may slip again. An algorithm (10) has been used to model this clutch stick-slip. In numerical simulations the algorithm determines the state of the system at every time step, the simulation is piecewise and non-linear. The state is determined by first checking the slip speed inequality and then if necessary checking the holding torque versus shearing torque inequality. The equations of motion for each state, sliding or engaged clutch (stichon) are then solved as dictated. Whilst sliding the direction of sliding dictates the sign of the clutch torque. If the system commences sliding from stichon, the sign of the clutch torque needs to be determined from the torque flow in the system.

T_C is a non-linear friction torque, a function of sliding velocity and the normal force:

$$T_C = \begin{cases} \operatorname{sgn}(\dot{\theta}_2 - \dot{\theta}_3) T_S & \text{if } \left|(\dot{\theta}_2 - \dot{\theta}_3)\right| \geq \varepsilon_{tol} \\ \operatorname{sgn}(T_{INT}) T_S & \text{if } \left|(\dot{\theta}_2 - \dot{\theta}_3)\right| < \varepsilon_{tol} \text{ and } T_{ST} < \left|T_{INT}\right| \\ T_{INT} & \text{if } \left|(\dot{\theta}_2 - \dot{\theta}_3)\right| < \varepsilon_{tol} \text{ and } T_{ST} \geq \left|T_{INT}\right| \end{cases} \qquad (10)$$

Where ε_{TOL} is the tolerance of zero velocity for numerical simulations and T_S and T_{ST} are clutch slipping and maximum holding torques. (dependent on kinetic and static friction respectively). They can be found with (8). The internal shear is more difficult to calculate and is found numerically [7].

Gear Lash – Contact Non-Linearity

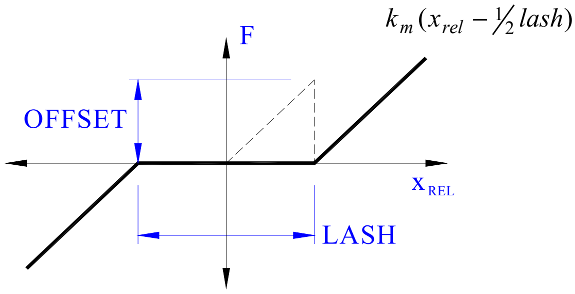
Gear lash is the clearance between mating gears that is required for their smooth rotation. In a loaded system meshing gears can lose contact during torque reversal. Within a certain range of lash the system is disconnected and the gear mesh stiffness is zero. When the gears reengage the mesh stiffness is a combination of tooth bending and material compression, the impact causes transient vibration and noise. This non-linear stiffness can be presented as restoring force as a function of relative position. The algorithm used to model the gear lash is:

For mesh stiffness:

$$k_3 = \begin{cases} k_m & if \ \ |(x_{rel})| \geq {lash}/{2} \\ 0 & if \ \ |(x_{rel})| < {lash}/{2} \end{cases} \qquad (11)$$

Where:

$$x_{rel} = r_4\theta_4 - r_5\theta_5 \qquad (12)$$



Non-linear gear mesh function

With the spring force, F, non-linear in this way the torque vector is needed to account for the offset of force from the zero position. The global torque vectors (5) and (7) are modified to include the torque offset:

$$T = \{T_E \quad T_C \quad -T_C \quad T_4 \quad T_5 \quad -T_V\} \text{ slipping} \qquad (13)$$

$$T = \{T_E \quad 0 \quad T_4 \quad T_5 \quad -T_V\} \text{ engaged} \qquad (14)$$

$$T_4 = -sign(x_{rel}) r_4 k_3 \, {lash}/{2} \qquad T_5 = sign(x_{rel}) r_5 k_3 \, {lash}/{2} \qquad (15)/(16)$$

When the stiffness, k_3, is found to be zero, the torque offset is returned as zero.

4.3 Simulation for clutch engagement

Numerical simulations have been programmed in Matlab for the powertrain system of figure 4. Results are given in the appendix: The driving torques in figure 6. Component speeds in figure 7. The shuffle transient from clutch engagement is shown in the axle shaft displacement (figure 8). Stick-slip occurs briefly from a high breakaway torque (figure 9) and high frequency transient vibration follow gear impacts from torque reversal (figure 10). For brevity simulation notes are given in bullets:

- Solves equation of motion (3) for clutch engagement transient dynamics
- Piecewise on engaged (5 DOF) and disengaged (6 DOF) states
- Equations of motion reduced to 1st order for use with Matlab's ODE15S solver
- Includes stick slip and gear lash algorithms which govern non-linear dynamics
- The systems external torques or forcing functions are the inputs into the model and define the shifting process

- Engine torque is interpolated from the engine torque map at 40% throttle with added harmonics for a typical 4 cylinder engine At 40% throttle the engine torque is fairly constant for the engine speeds in this simulation
- Clutch pressure is already applied yielding 200Nm torque and increases linearly with time with added oscillations at 45hz. The oscillations are not modelled from a real hydraulic system or determined from clutch judder but are used to easily induce the system response needed to demonstrate the stick-slip phenomena. Both the hydraulic system and the friction characteristics can induce torque oscillations in real systems
- Vehicle aerodynamic drag, rolling resistance and gradient torques are combined and set to a constant value of 100Nm
- Damping is minimal to provide larger transients to easier illustrate the stick-slip and gear lash
- Gear lash is set at 0.5mm
- The tolerance of relative velocity for the stick-slip algorithm is set at ±0.01 rad/s
- The initial conditions for the simulation provide balanced spring torque
- Time stepping reduced around stick point and lash
- Offsets removed for absolute displacement and velocity on piecewise transition
- Care taken when using tolerance of zero velocity

5.0 EXPERIMENTAL VALIDATION

Test rigs are used for powertrain tuning and research. Arrangements range from an engine or tire test device to a whole powertrain with added vehicle load and equivalent mass. For dynamics, typical test rig uses include, investigating component characteristics (engine, torque converter, clutch, tire, etc.), investigating free, steady state and transient response and calibration for gear shifts.

At the University of Technology, Sydney a powertrain test rig has been constructed for the investigation of vibration response and gear shift quality assessment. The aim is to verify dynamic analysis using a model similar to that of figure 2. The dynamic model includes gear lash for planetary gears and for the final drive ring, pinion, and differential gears. It is used for free vibration analysis and steady state and transient numerical simulations.

The test rig includes all the components of the vehicle powertrain and has been designed to include a vehicle mass of 1500 kg. This is achieved by replacing the approximate mass of the vehicle with and equivalent inertia. A combination of small and large train wheels have been used to provide the correct inertia and are mounted so as to be driven by car tires. A linear to rotational kinetic energy relationship is applied to solve the kinetic energy equation for equivalent inertia. The load from aerodynamic drag and any road gradient has been introduced to the test rig via a dynamometer. The load is applied to the flywheels via another final drive and tire subsystem. The front final drive ratio is 3.1:1 and the rear 5:1. The coupling from the flywheels to the dynamometer increases velocity at the dynamometer to within its operating range. For tire speed from 20-100 km/hr dynamometer speed is 850-4300 rpm.

For data acquisition the engine and transmission control systems are tapped and instrumentation added for pressures, torques and accelerations. Accelerometers are fixed on the transmission and differential case.

For the propeller shaft and tire they are demountable as they are used in free vibration tests only, also their linear accelerations are converted to angular. Torque is measured via strain gauges on the flywheel, transmission output shaft and drive shaft. Radio telemetry is used to pass the strain gauge data from the rotating shaft to a non-rotating element. Another method is to use slip rings. The gauge voltage is amplified, processed by an analogue to digital converter and then transmitted. Transceivers are used on both rotating and non-rotating sides. Data is recorded and post processed with Lab View. The dynamic measurements include:

- Engine output torque and angular velocity
- Transmission output torque and angular velocity
- Drive shaft torque and angular velocity
- Transmission case and differential case linear acceleration
- Propeller shaft and tire angular acceleration
- Sump temperature
- Shift schedule and throttle position
- Air inlet pressure
- Clutch and band applied pressures

Parameters for inertia and stiffness and gear dynamics are determined from detail drawings and measurements. Parameters that are difficult to determine include tire stiffness, clutch and band friction characteristics, torque converter output torque and system damping – to name some. For the test rig and dynamic model to be compared both the system parameters and forcing functions must be the same. The measured engine torque, clutch pressure (torque) and vehicle load are to be used in numerical simulations as driving functions. The measured output and drive shaft torque and case accelerometers are used to compare with numerical simulation results. Various tests can be conducted with this test rig:

<u>Free vibration</u>

Test: The transmission is placed in park (grounding the rigid body motion). A torque is applied to the tires and released. Accelerometers and torque sensors provide free vibration results.

Purpose: To compare real system frequency response to free vibration analysis of the driveline system. This allows a validity check for the stiffness and inertia parameters and driveline dynamic model.

<u>Critical Speed</u>

Test: The engine is run within speed ranges that are calculated for resonance for given gear states. Shaft torque and case accelerations provide steady state response at test speeds.

Purpose: To compare resonant modes for the powertrain system. This allows a validity check for the stiffness, inertia and damping parameters and the whole dynamic model.

<u>Engine Tip in/out</u>

Test: The engine is run at a constant speed and the throttle is suddenly increased or decreased. Shaft torque and case accelerations provide transient response.

Purpose: To investigate driveline shuffle (lowest global mode) and clonk (backlash). This allows a validity check for the stiffness, inertia and damping parameters and the whole dynamic model. Case accelerometers should indicate high frequency transients from gear backlash.

Gear Shifting

Test: Gear shifts are performed for various throttle settings. Shaft torque and case accelerations provide transient response.

Purpose: To investigate transient torque from gear shifts and associated driveline shuffle as well as oscillations at higher modes. This allows a validity check for gearshift numerical simulations. Case accelerometers should indicate high frequency transients from any gear backlash.

Where possible tests will be run with the torque converter lock up clutch engaged. This is as the empirical parameters required for a torque converter numerical model are unavailable. Measuring the output torque of the torque converter within a vehicle test rig is a significant challenge.

6.0 CONCLUSIONS

The principle aim of this report was to illustrate an approach to modelling, simulation and experiments used for investigating powertrain torsional vibration. For *modelling*, how the finite element method can be used for various powertrains to detailed levels is illustrated (figures 2 and 3). Custom elements can be developed for planetary gear dynamics with rigid [3] or elastic tooth mesh and in the same fashion for final drive dynamics. For *numerical simulation*, gear lash elements can be introduced to these dynamics [8]; for both a two stage planetary gear set and a conventional final drive 6x6 custom finite elements can be used with elastic non-linear tooth mesh (to be reported). Transient simulations can be used for tip-in and gear shifting with stick-slip and rattle. For gear shifting, apply torque equations with experimentally determined friction coefficients improve judder and stick-slip modelling [7]. The method can be used for manual, automatic transmission and CVT powertrains. An example was used with a model to investigate clutch stick-slip and gear rattle to help illustrate the dynamic modelling and the simulation techniques. Results are provided but not detailed due to space constraints. The modelling and simulation can be extended further than already noted. The various components of the powertrain bring complexity and are non-linear; the engine can be modelled or measured for combustion torque, the torque converter for transfer torque, bands and clutch for friction characteristics, judder and stick-slip, hydraulics for pressure, gear dynamics for gear shifts, gear meshing for backlash and transmission error, tires for damping, stiffness and slip and then the associated systems – electrical, control, chassis and suspension. For *testing*, a rig designed to simulate a vehicle powertrain was detailed with notes on instrumentation. Discussion relates to testing for the verification of dynamic modelling and analytical and numerical analysis.

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APPENDICIES

$$I_{e(n+1)} = \begin{bmatrix} J_n & 0 \\ 0 & J_{n+1} \end{bmatrix} \quad K_{e(n+1)} = \begin{bmatrix} k_{n+1} & k_{n+1} \\ k_{n+1} & k_{n+1} \end{bmatrix} \quad C_{e(n+1)} = \begin{bmatrix} c_{n+1} & -c_{n+1} \\ -c_{n+1} & c_{n+1} \end{bmatrix} \quad \theta_{e(n+1)} = \begin{Bmatrix} \theta_n \\ \theta_{n+1} \end{Bmatrix} \qquad (17)$$

$$I_{e(n+1)} = \begin{bmatrix} n_G^2 J'_n & 0 \\ 0 & J_{n+1} \end{bmatrix} \quad K_{e(n+1)} = \begin{bmatrix} n_G^2 k_{n+1} & -n_G k_{n+1} \\ -n_G k_{n+1} & k_{n+1} \end{bmatrix} \quad C_{e(n+1)} = \begin{bmatrix} n_G^2 c_{n+1} & -n_G c_{n+1} \\ -n_G c_{n+1} & c_{n+1} \end{bmatrix} \quad \theta_{e(n+1)} = \begin{Bmatrix} \theta_n \\ \theta_{n+1} \end{Bmatrix} \qquad (18)$$

$$I_{e(n+1)} = \begin{bmatrix} J_n & 0 \\ 0 & J_{n+1} \end{bmatrix} \quad K_{e(n+1)} = \begin{bmatrix} r_n{}^2 k_{n+1} & -r_n r_{n+1} k_{n+1} \\ -r_n r_{n+1} k_{n+1} & r_{n+1}{}^2 k_{n+1} \end{bmatrix} \quad \theta_{e(n+1)} = \begin{Bmatrix} \theta_n \\ \theta_{n+1} \end{Bmatrix} \qquad (19)$$

$$I_{e(n+1)} = \begin{bmatrix} \frac{J_n}{2} & 0 \\ 0 & J_{n+1} \end{bmatrix} \quad K_{e(n+1)} = \begin{bmatrix} k_{n+1} & -k_{n+1} \\ -k_{n+1} & k_{n+1} \end{bmatrix} \quad C_{e(n+1)} = \begin{bmatrix} c_{n+1} & -c_{n+1} \\ -c_{n+1} & c_{n+1} \end{bmatrix} \quad \theta_{e(n+1)} = \begin{Bmatrix} \theta_n \\ \theta_{n+1} \end{Bmatrix} \qquad (20A)$$

$$I_{e(n+2)} = \begin{bmatrix} \frac{J_n}{2} & 0 \\ 0 & J_{n+2} \end{bmatrix} \quad K_{e(n+2)} = \begin{bmatrix} k_{n+2} & -k_{n+2} \\ -k_{n+2} & k_{n+2} \end{bmatrix} \quad C_{e(n+2)} = \begin{bmatrix} c_{n+2} & -c_{n+2} \\ -c_{n+2} & c_{n+2} \end{bmatrix} \quad \theta_{e(n+2)} = \begin{Bmatrix} \theta_n \\ \theta_{n+2} \end{Bmatrix} \qquad (21B)$$

$$I_{e(n)} = [J_n] \quad K_{e(n)} = [k_n] \quad C_{e(n)} = [c_n] \quad \theta_{e(n)} = \{\theta_n\} \qquad (22)$$

Table 1. Torsional Finite Elements for Direct, Rigid Geared, Elastic Geared, Branched and Grounded Systems

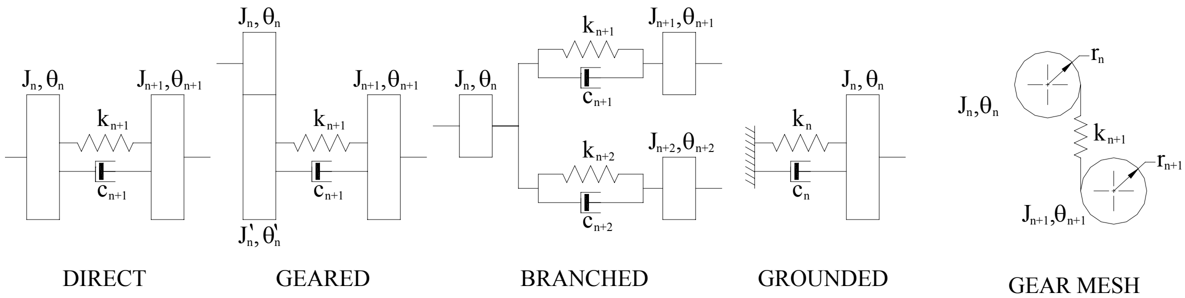

Figure 1. Model Schematics

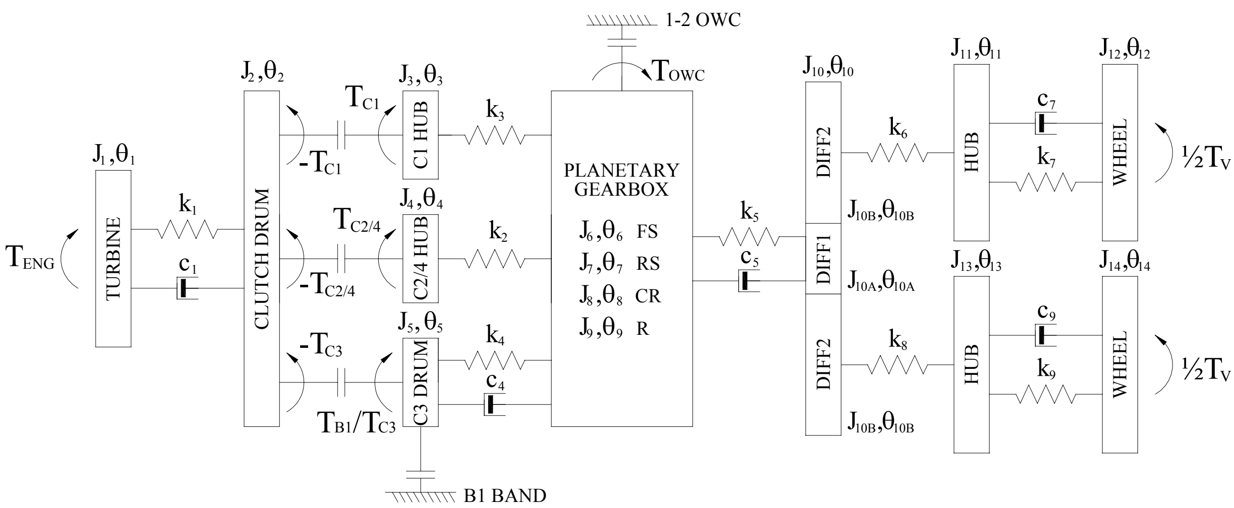

Figure 2. Dynamic Model for Powertrain System with Automatic Transmission

$$\mathrm{I}_{e2} = \begin{bmatrix} J_2 & 0 \\ 0 & \frac{J_3}{2} \end{bmatrix} \quad \mathrm{K}_{e2} = \begin{bmatrix} n_2{}^2 k_2' & -n_2 k_2' \\ -n_2 k_2' & k_2' \end{bmatrix} \quad \mathrm{C}_{e2} = \begin{bmatrix} n_2{}^2 c_2' & -n_2 c_2' \\ -n_2 c_2' & c_2' \end{bmatrix} \quad \theta_{e2} = \begin{Bmatrix} \theta_2 \\ \theta_3 \end{Bmatrix} \quad n_2 = \frac{r_2}{r_3} \tag{23}$$

$$\mathrm{I}_{e3} = \begin{bmatrix} \frac{J_3}{2} & 0 \\ 0 & J_4 \end{bmatrix} \quad \mathrm{K}_{e3} = \begin{bmatrix} n_3{}^2 k_3' & -n_3 k_3' \\ -n_3 k_3' & k_3' \end{bmatrix} \quad \mathrm{C}_{e3} = \begin{bmatrix} n_3{}^2 c_3' & -n_3 c_3' \\ -n_3 c_3' & c_3' \end{bmatrix} \quad \theta_{e3} = \begin{Bmatrix} \theta_3 \\ \theta_4 \end{Bmatrix} \quad n_3 = \frac{r_3}{r_4} \tag{24}$$

Table 2. Torsional Finite Elements for Toriod-Roller Contact

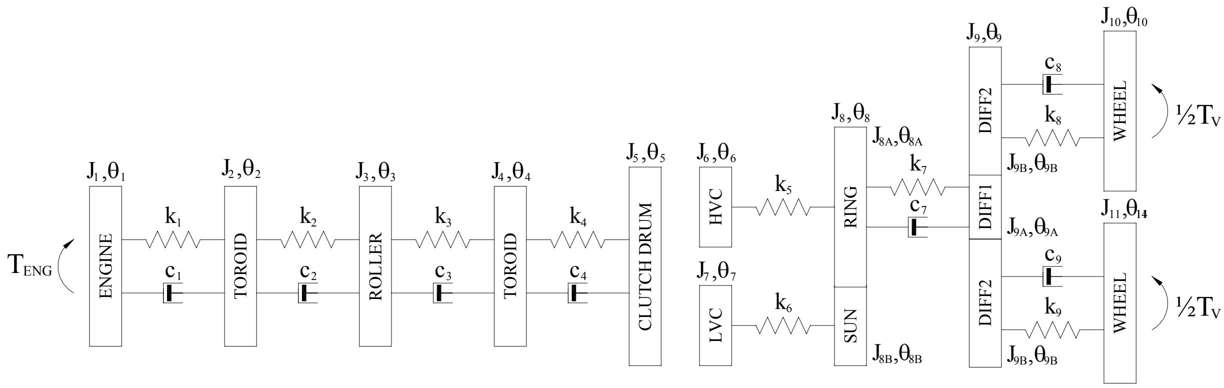

Figure 3. Dynamic Model for Powertrain Fitted with CVT and Planetary Gear Set

clutch slipping | clutch engaged

$$\mathrm{I}_{e1} = \begin{bmatrix} J_1 & 0 \\ 0 & J_2 \end{bmatrix} \quad \mathrm{K}_{e1} = \begin{bmatrix} k_1 & -k_1 \\ -k_1 & k_1 \end{bmatrix} \quad \mathrm{C}_{e1} = \begin{bmatrix} c_1 & -c_1 \\ -c_1 & c_1 \end{bmatrix} \quad \theta_{e1} = \begin{Bmatrix} \theta_1 \\ \theta_2 \end{Bmatrix} \quad \theta_{e1} = \begin{Bmatrix} \theta_1 \\ \theta_{2,3} \end{Bmatrix} \tag{25}$$

$$\mathrm{I}_{e2} = \begin{bmatrix} J_3 & 0 \\ 0 & 0 \end{bmatrix} \quad \mathrm{K}_{e2} = \begin{bmatrix} k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} \quad \mathrm{C}_{e2} = \begin{bmatrix} c_2 & -c_2 \\ -c_2 & c_2 \end{bmatrix} \quad \theta_{e2} = \begin{Bmatrix} \theta_3 \\ \theta_4 \end{Bmatrix} \quad \theta_{e2} = \begin{Bmatrix} \theta_{2,3} \\ \theta_4 \end{Bmatrix} \tag{26}$$

$$\mathrm{I}_{e3} = \begin{bmatrix} J_4 & 0 \\ 0 & J_5 \end{bmatrix} \quad \mathrm{K}_{e3} = \begin{bmatrix} r_4^2 k_3 & -r_4 r_5 k_3 \\ -r_4 r_5 k_3 & r_5^2 k_3 \end{bmatrix} \quad \theta_{e3} = \begin{Bmatrix} \theta_4 \\ \theta_5 \end{Bmatrix} \quad \theta_{e3} = \begin{Bmatrix} \theta_4 \\ \theta_5 \end{Bmatrix} \tag{27}$$

$$\mathrm{I}_{e4} = \begin{bmatrix} 0 & 0 \\ 0 & J_6 \end{bmatrix} \quad \mathrm{K}_{e4} = \begin{bmatrix} k_4 & -k_4 \\ -k_4 & k_4 \end{bmatrix} \quad \mathrm{C}_{e4} = \begin{bmatrix} c_4 & -c_4 \\ -c_4 & c_4 \end{bmatrix} \quad \theta_{e4} = \begin{Bmatrix} \theta_5 \\ \theta_6 \end{Bmatrix} \quad \theta_{e4} = \begin{Bmatrix} \theta_5 \\ \theta_6 \end{Bmatrix} \tag{28}$$

Table 3. Torsional Finite Elements for Powertrain Dynamic Model with Clutch Engagement

Parameter	Value
J_1	0.25 kgm²
J_2	0.001 kgm²
J_3	0.001 kgm²
J_4	0.005 kgm²
J_5	0.015 kgm²
J_6	70 kgm²
k_1	10000 Nm/rad
k_2	30000 Nm/rad
k3/km	1e8 Nm
k4	10000 Nm/rad
c1	2 Nms/rad
c2	.5 Nms/rad
c4	4 Nms/rad
r4	0.0218 m
r5	0.0675 m
r5	0.0005 m

Table 4. System parameters

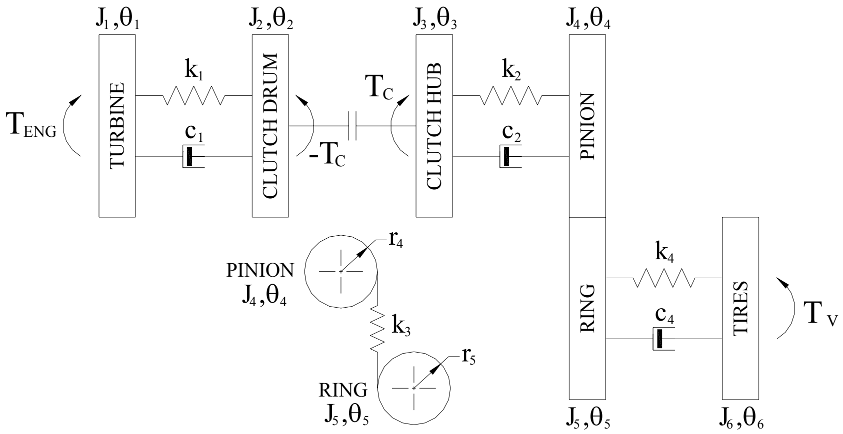

Figure 4. Powertrain Dynamic Model with Clutch Engagement

Frequency (Hz)	1031	394	106	9.31	0
Engine	0.00	0.00	-0.09	-1.00	1.00
Clutch	-0.04	-0.36	0.94	-0.91	1.00
Pinion	1.00	1.00	1.00	-0.88	1.00
Ring	-0.89	0.39	0.32	-0.28	0.32
Tires	0.00	0.00	0.00	0.01	0.32
Damped Frequency (Hz)	1017	388	104	9.20	
damping ratio	1.70%	3.7%	7.2%	1.1%	

Figure 5. Engaged System Natural Modes and Damped Frequencies

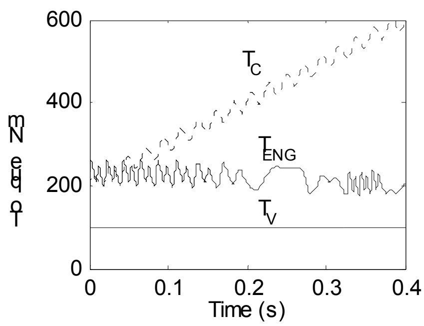

Figure 6. Engine, Clutch and Vehicle Torques

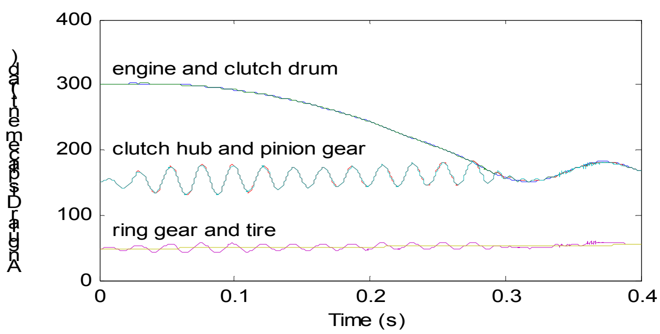

Figure 7. Angular Velocities of Powertrain Components

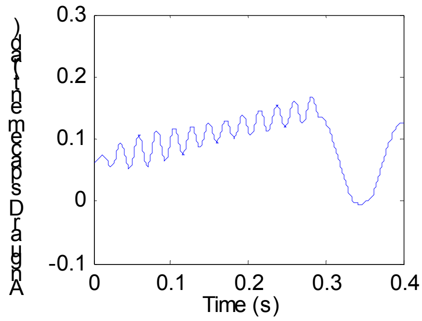

Figure 8. Relative Angular Displacement across axle shaft

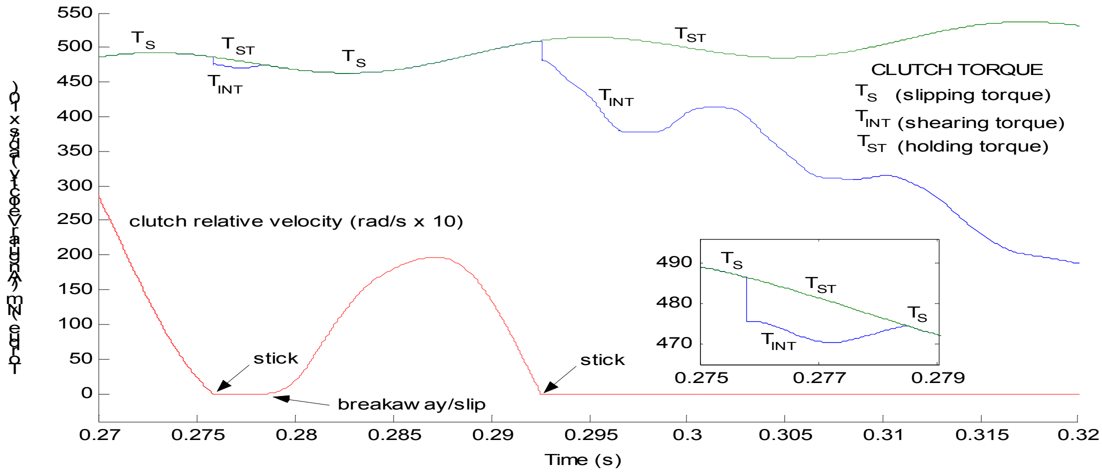

Figure 9. Clutch torques and relative velocity around clutch engagement point

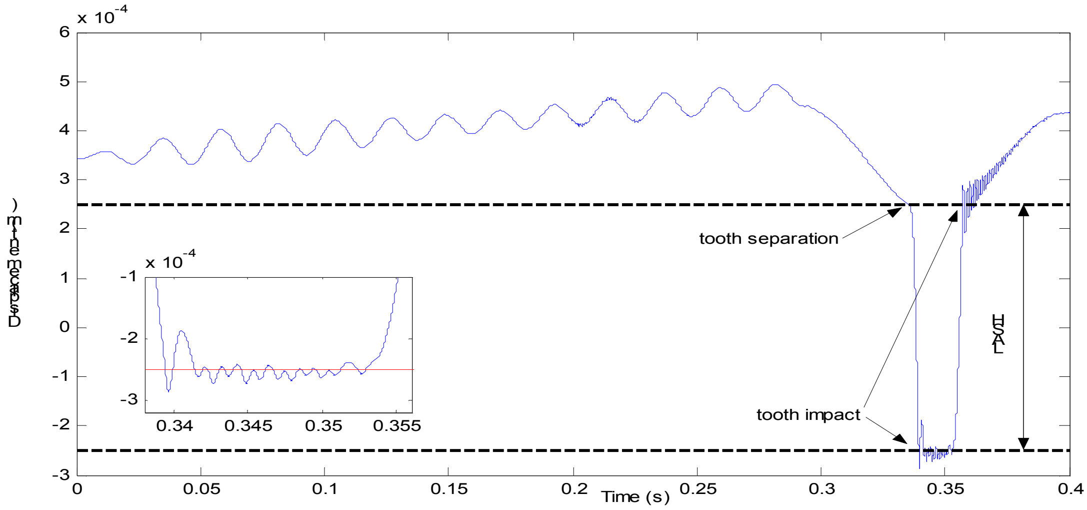

Figure 10. Ring and pinion tooth mesh relative displacement

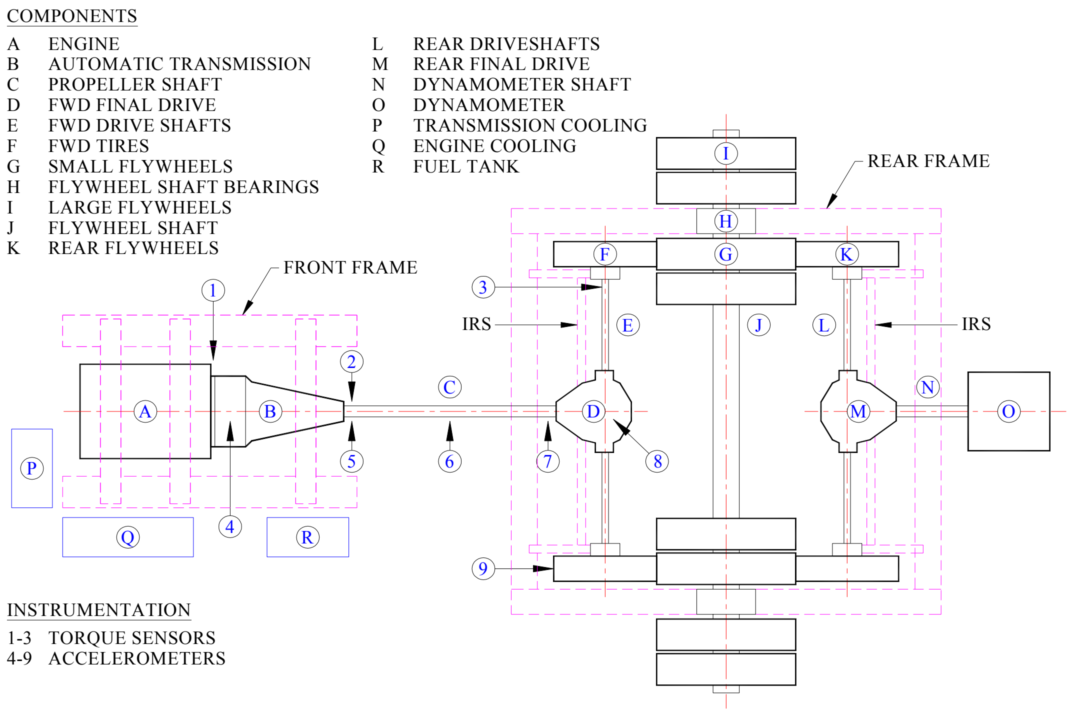

Figure 11. Powertrain Test Rig Schematic

REFERENCES

(1) J.-S.Wu, C.-H.Chen, Torsional Vibration Analysis of Gear-Branched Systems by Finite Element Method. .Journal of Sound and Vibration 240(1) 159-182, 2001

(2) Ph.Couderc, J.Callenaere, J.Der Hagopian, G.Ferraris, "Vehicle Driveline Dynamic Behaviour: Experiment and Simulation". Journal of Sound and Vibration 218(1) 133-157, 1998

(3) N.Zhang, A.Crowther, D.K.Liu, J.Jeyakumaran, "A Finite Element Method for the Dynamic Analysis of Automatic Transmission Gear Shifting with a 4DOF Planetary Gearset Element". Proc. of Inst. of Mech. Eng. Part D: Journal of Automobile Engineering 217 461-473, 2003

(4) W.Reik, "Torsional Vibrations in the Drive Train of Motor Vehicles – Principle Considerations". Proceedings of the 4th International Symposium on Torsional Vibrations in the Drivetrain Baden-Baden, Gernmany, 20 April 1990

(5) S.-J.Hwang, J.-S.Chen, L.Liu, C.-C.Ling, "Modeling and Simulation of a Powertrain Vehicle System with Automatic Transmission". International Journal of Vehicle Design 23 (1) 145-160, 2000

(6) A.Farshidianfar, A.Ebrahimi, H.Bartlett, "Hybrid Modelling and Simulation of the Torsional Vibration of Driveline Systems". Proc. of Inst. of Mech. Eng. Part D: Journal of Automobile Engineering 215 217-229, 1999

(7) A. Crowther, N. Zhang, D. Liu, J.M. Jeyakumaran, "Analysis and Simulation of Clutch Engagement Judder and Stick-Slip in Automotive Powertrain Systems" Proc. of Inst. of Mech. Eng. Part D: Journal of Automobile Engineering (submitted) 2004

(8) A. Crowther, N. Zhang, D. Liu, J.M. Jeyakumaran, "Torsional Finite Elements And Non-Linear Numerical Modelling In Vehicle Powertrain Dynamics" Journal of Sound and Vibration (submitted) 2004

(9) J.W.Biermann, B.Hagerodt, "Investigation of the Clonk Phenomenon in Vehicle Transmissions – Measurement, Modeling and Simulation". Proc. of Inst. of Mech. Eng. Part K: Journal of Multibody Dynamics, 213 53-60, 1999

(10) J.A.C. Martins, J.T. Oden, F.M.F. Simoes, "Recent Advances in Engineering Science – A Study of Static and Kinetic Friction". Int. Journal of Engineering Science 28(1) 29-92, 1990

(11) H.Kohler, Gear Dynamics – Studies of 'The State of the Art' No. 2 British Gear Association Design Sub-Committee, December 1998