

**TECHNICAL PAPER FOR STUDENTS AND YOUNG ENGINEERS**  
**- FISITA WORLD AUTOMOTIVE CONGRESS, BARCELONA 2004 -**

TITLE:

**PARAMETRIC MULTI-BODY ANALYSIS OF KART DYNAMICS**

Topic:

- FUTURE AUTOMOTIVE TECHNOLOGY       INTELLIGENT TRANSPORTATION SYSTEMS  
 USER FRIENDLY AUTOMOBILE       ADVANCED PRODUCTION AND LOGISTICS  
 VEHICLES & THE ENVIRONMENT

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National Society:

YES       NO

Name of the National Society:

In this work a multi-body analysis of kart dynamics has been performed using the commercial software Working Model. For this purpose the vehicle has been modelled using a lumped parameters model consisting of three rigid body linked by two elastic elements, that represent the frame torsional stiffness. In this way static and dynamic load transfers can be evaluated. The model also considers the particular geometry of the steering system, and also allows to change the characteristic parameters ( i.e. caster and king-pin angles, length of the lever arms and tie rods, etc) in order to appreciate the changes in vehicle performance.

This work also takes into account a tyre model in order to represent the cornering forces, using a model based on an exponential formulation obtained by fitting experimental data.

Place / Date:

Rome, 16/02/04

## 1. INTRODUCTION

Competition go karts are a very particular kind of vehicles. Their principal characteristic is the extreme constructive simplicity but at the same time, their design and tuning can be more complex than that of a standard vehicle. In fact the absence of differential gear and of any kind of suspension system, the asymmetric position of pilot and engine, and the particular configuration of the braking system makes frame shape and stiffness and setup parameters (i.e. caster angle, geometry of steering system, diameter of rear axle, etc.) to play a very important role in vehicle behaviour.

Until today karts have been developed principally in an empiric way but, likewise it happens in other formulas, they can take all the advantages of numerical simulation to improve performances and reduce lap time.

In this paper a multy-body analysis of kart dynamics is presented. The model, trimmed according to experimental data is able to simulate any kind of manoeuvre.

In particular, in this work a double change of trajectory, based on experimental results, is realized, and the trend of the principal dynamic parameters (i.e. cornering angles, lateral acceleration and forces, etc.) is presented.

Simulation of a constant radius corner covered at various speed is developed too, in order to analyze the influence of set up parameters in kart's performance and evaluate the handling diagram of vehicle.

## 2. DESCRIPTION OF KART MODEL

### 2.1 FRAME MODEL

Frame is described by a lumped parameter schematization made up of three rigid bodies joined by two torsional springs with damper ( $K_f$ ,  $K_r$ ), representing frame torsional stiffness. In this way it is possible to evaluate static and dynamic load transfers acting on tyres.

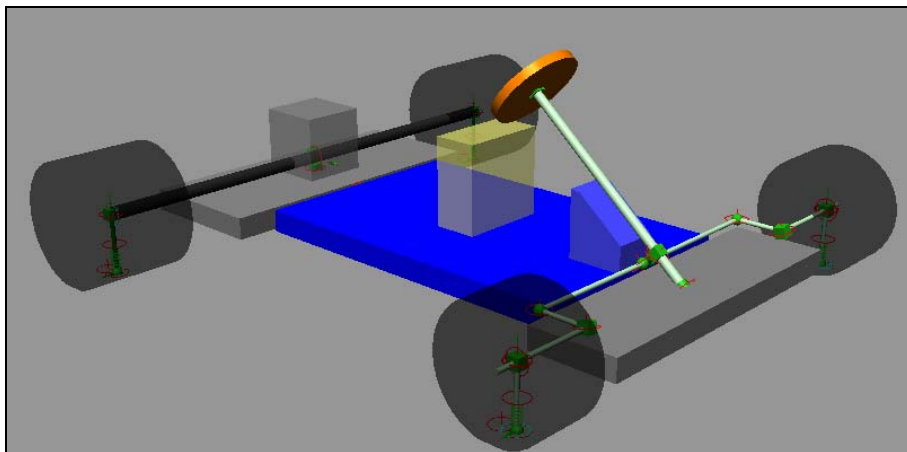


Figure 1: Kart multy-body model

The values for spring's torsional stiffness were taken by technical literature [3] and they consist of :

$$K_f = 240.4 \frac{Nm}{deg} \quad K_r = 533 \frac{Nm}{deg}$$

respectively for front axle and rear axle; moreover they respect the equivalence with the FEM frame model developed in [2] and [5] in which a torsional stiffness of 164 Nm/deg is evaluated. The correctness of assumed data is shown by the accordance of the model with the experimental data measured on track (see par. 3)

The shape of the three rigid bodies is chosen in order to obtain the same natural frequencies of FEM frame model (i.e.  $f_1=46$  Hz;  $f_2=210$ Hz), in fact being rotational stiffnesses and masses (12 kg for the frame) already fixed, the torsional modes depend only by rotational inertia along the longitudinal axis of vehicle. In this way section's dimension of bodies is determined, and a more realistic dynamic behaviour can be simulated.

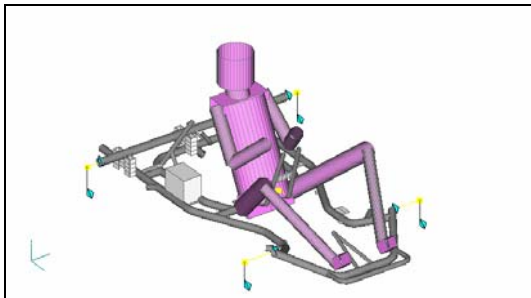


Figure 2: Complete FEM model of frame

|                      |        |
|----------------------|--------|
| Pilot                | 70 Kg  |
| Front axle           | 2 Kg   |
| Rear axle            | 4 Kg   |
| Central body         | 6 Kg   |
| Engine, conducts etc | 31 Kg  |
| Fuel Tank, bumpers   | 16 Kg  |
| TOTAL                | 154 Kg |

Tab1: Mass distribution of kart model

Moreover other masses were added in order to represent all the components and obviously to respect total kart's mass with pilot. The distribution is shown in Tab 1.

## 2.2 STEERING SYSTEM MODEL

Steering system's geometry was fully modelled in order to reproduce the vertical translation of front wheels as a function of the steering column angle. Such translation is due to the mounting configuration of the stub axle that is not perpendicular to the ground, but inclined along a direction defined by caster and king-pin angles.

This is very important in kart's behaviour: wheel displacement, in fact, and a good shaped frame, usefully help internal rear wheel to rise during the main part of a curve, avoiding tire slip due to the absence of the differential gear.

In the model caster and king-pin angle were set equal to  $14^\circ$  and  $10^\circ$ , while other dimensions are shown in figure 3.

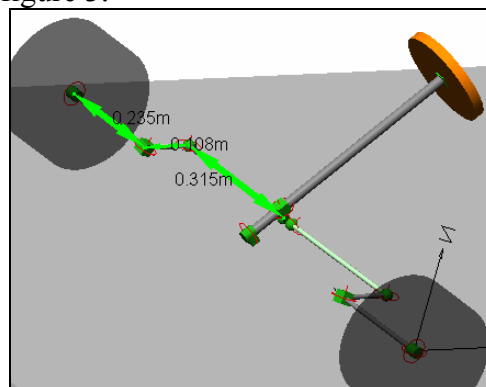


Figure 3: Kart steering system.

Figure 4 shows front wheels displacement (a) and the stubs angles (b) as a function of the steering column angle.

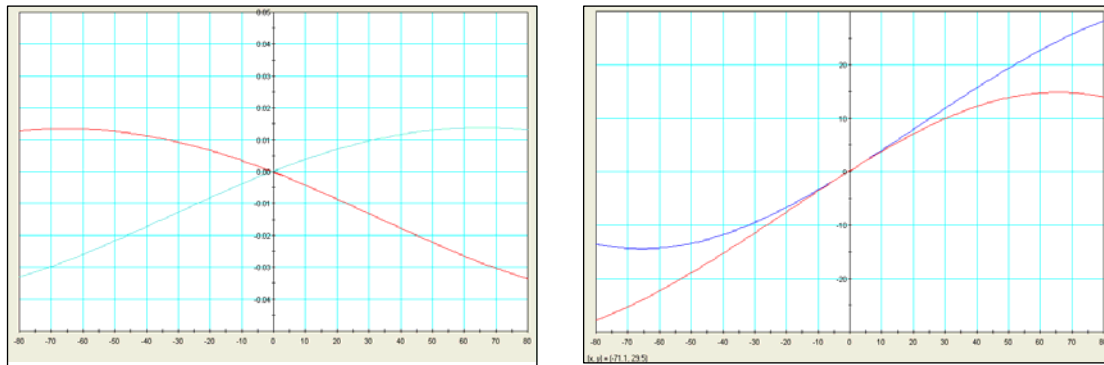


Figure 4: (a) Wheel displacement as a function of steering angle; (b) stub angles as a function of steering angle.

### 2.3 TIRE MODEL

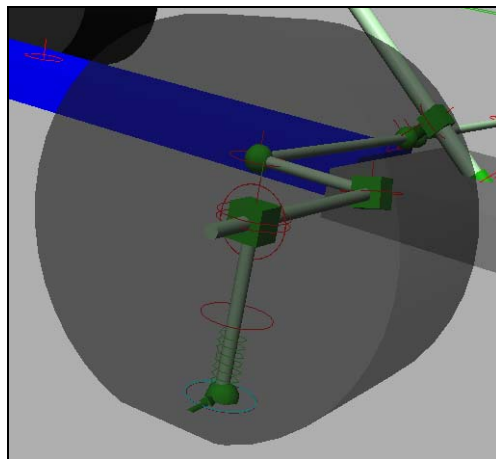


Figure 5: Tyre model.

Tire contact was modelled by a rod connected with the hub by a *rigid joint on slot* constraint and with ground by a spring-damper system in which the elastic constant of the spring represents tire radial stiffness and is taken (in according with experimental data in [2],[6]) as 75 KN/m for front tires and 100 KN/m for rear tires. Damping is calculate as follows:

$$\omega_n = \sqrt{\frac{k}{m}}, \quad c = 2\zeta\omega_n$$

Where  $k$  stands for the radial tire stiffness,  $m$  is kart's mass acting on the tire,  $\omega_n$  is tire's natural circular frequency and  $\zeta = 0.2$  is damping vs. critic damping ratio.

Contact point with ground is simulated using a *generic constraint* which forces constraint position to be greater or equal than zero. Anyway the wheel was modelled by means of a cylinder rigidly connected to the hub in order to have the proper look and mass distribution

Tyre model also takes into account the presence of cornering forces implemented on the basis of the formulation described in [3] (which regards a single track model) and rearranged in [4] for a four wheel model:

$$F_{yiF} = \frac{F_{f \max}}{2} \frac{F_z}{F_z^*} \left( 1 - e^{-\frac{\alpha_f F_z^*}{\alpha_{f \max} F_z}} \right);$$

$$F_{yiR} = \frac{F_{r \max}}{2} \frac{F_z}{F_z^*} \left( 1 - e^{-\frac{\alpha_r F_z^*}{\alpha_{r \max} F_z}} \right)$$

The constants that appears in the formula are measured in [6] and assumes the following values:

$$\alpha_{f \max} = 5.4; \quad \alpha_{r \max} = 5; \quad F_{f \max} = 900; \quad F_{r \max} = 1300.$$

where  $F_z$  is the load acting on the tire, while  $F_z^*$  stands for the mean value of the force acting on left and right wheels,  $\alpha$  is the cornering angle.

The longitudinal forces are simulated by a concentrated force on the mean of rear axle with an attractor on desired velocity:

$$F_m = k * \left( \frac{vd - va}{vd} \right)$$

Where  $vd = \omega * r_e$  is the desired velocity,( i.e. the product between the angular velocity of axle and the rolling radius),  $va$  is the real velocity and  $k$  is the gain

### 3. ACQUISITION OF EXPERIMENTAL DATA

The multibody model was statically validated on the basis of acquired data on track. In fact the kart (with pilot) was positioned on a plane and vertical load transfers at different steering 's degrees were measured by four load cells. In this way the effect of the caster and king-pin angle on load transfers can be analyzed.



**Figure 6: Static test for load transfers**

After measuring the steering system, tyres radius, front and rear tracks, the test was virtually executed by the model, and figure 6 shows how it was able to reproduce the real load transfers.

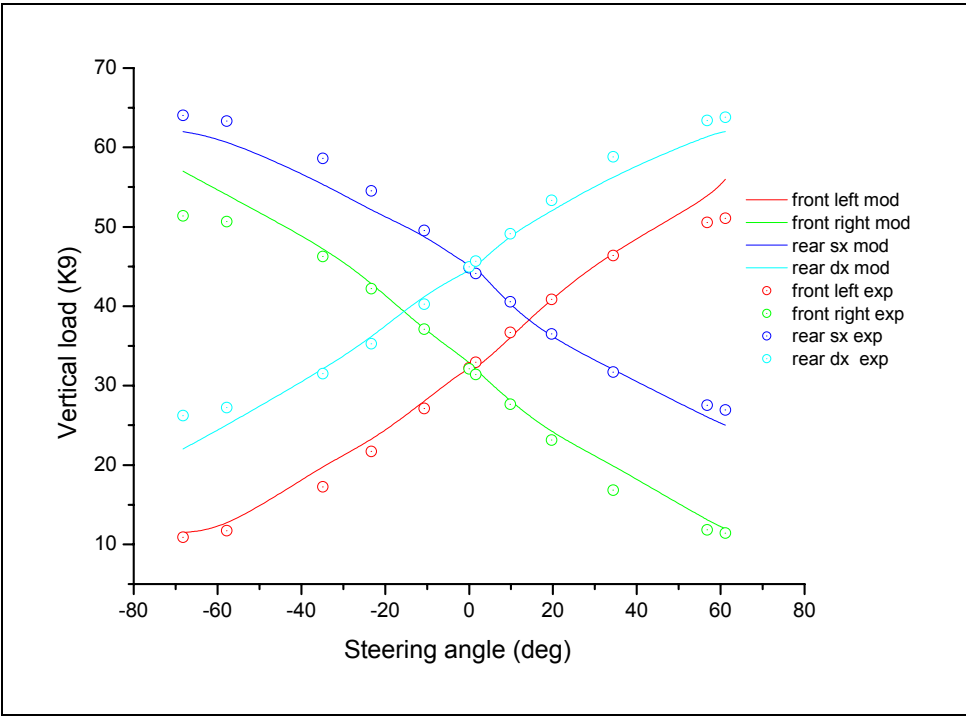


Figure 7: Wheels vertical loads, simulated and experimental, as a function of steering angle

In order to validate dynamically the kart model other data have been acquired, with a data acquisition system (AIM Evo3). Particularly the steering angle, longitudinal and lateral acceleration, kart’s angular velocity, wheel velocity, and engine rpm was measured during some manoeuvre ( as double change of trajectory and steering pad) and during track laps.

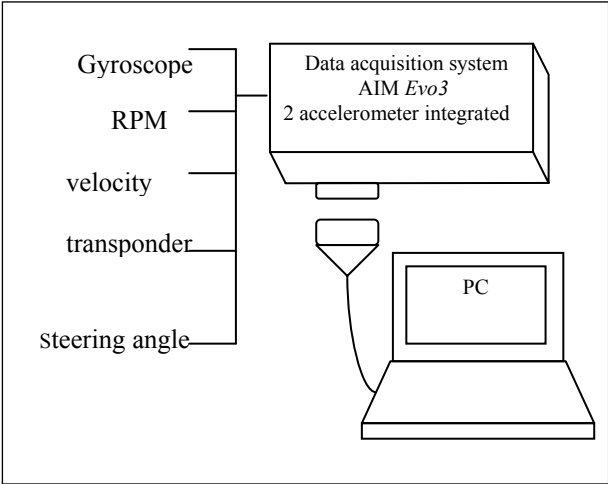
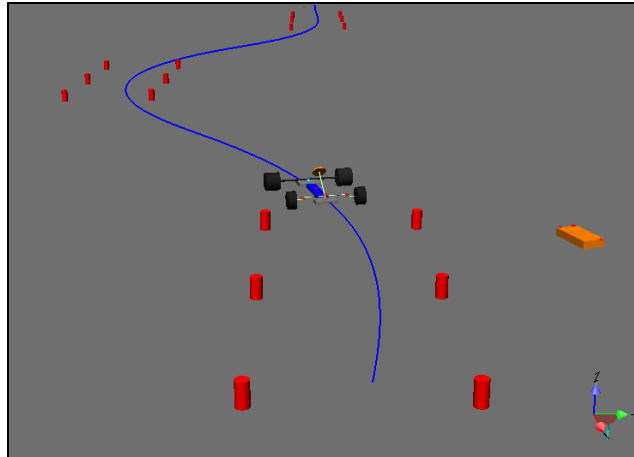


Figure 8: Data acquisition system

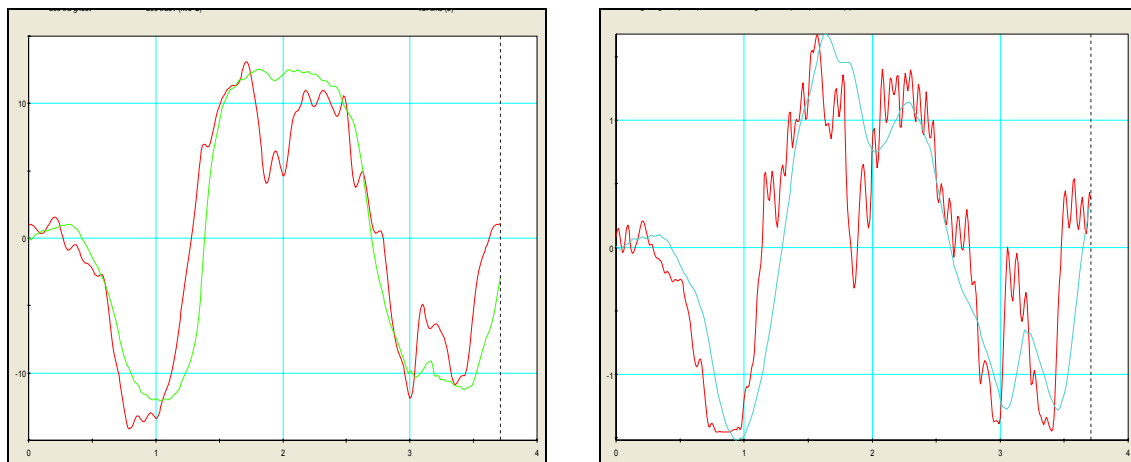
## 4. SIMULATIONS

The dynamical validation of kart model was reached simulating a double change of trajectory ( as ISO 3888 prescribes) ; measured values, after filtering, were introduced in multy-body model in order to reproduce the manoeuvre, and to compare simulated and experimental data. The last one were assigned to a ghost body (the orange one in fig9).



**Figure 9 :Simulation of double change of trajectory**

Figure 10 shows the comparison between lateral acceleration simulated and acquired (a) and yaw rate simulated and acquired (b).



**Figures 10 : (a) lateral acceleration simulated and acquired ; (b) yaw rate simulated and acquired.**

Moreover a constant radius curve covered at various velocities was simulated (figure11 a); in this way it is possible to analyze loads transfers on tires, longitudinal and lateral accelerations, cornering forces and angles, and the handling diagrams that show the under/over steering behaviour of kart.(figure 11 b). To simulate this manoeuvre a simple trajectory control acting on steering's angular velocity was implemented.

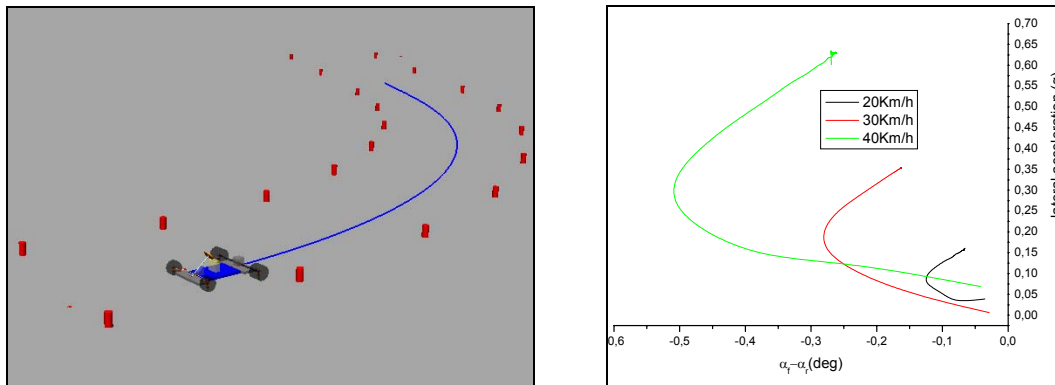


Figure 11: (a) 20m radius curve covered at 3 velocities; (b) handling diagram

## 6. CONCLUSIONS

In this work a multi-body model for competition go-kart was realized. The comparison with experimental acquired data allowed its static and dynamical validation; in this way it was possible to reproduce virtually some representative tests on the vehicle; thus this model presents the possibility to appreciate the change of vehicle performance during simulations, varying the characteristic parameters. Nevertheless the model contains significant simplifications such as the longitudinal tire's model and no interaction between lateral and longitudinal forces; for this purpose other data acquisitions will be programmed in order to develop these aspects.

## 7. ACKNOWLEDGMENTS

The authors would like to express their acknowledgments to Prof. M.E. Biancolini, Eng. R. Baudille and Eng. L. Reccia for their support and suggestions.

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